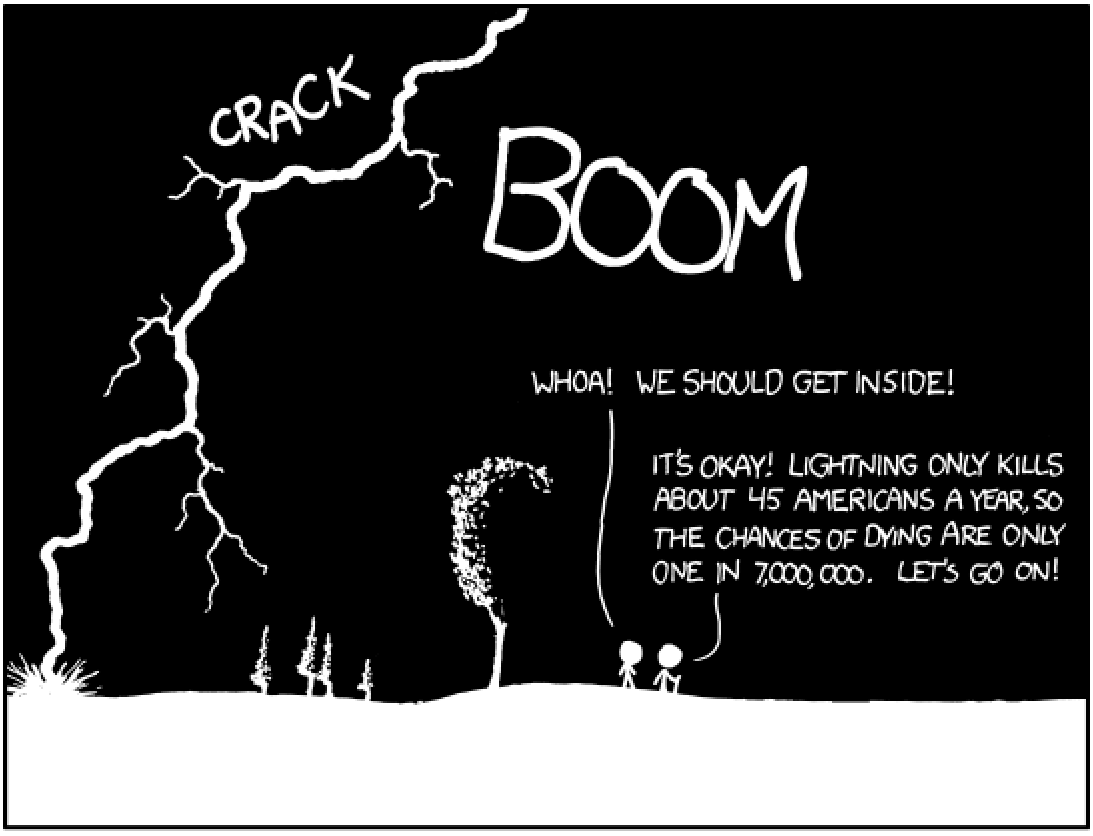
Probability



THE ANNUAL DEATH RATE AMONG PEOPLE WHO KNOW THAT STATISTIC IS ONE IN SIX.

Understanding theoretical and experimental probability

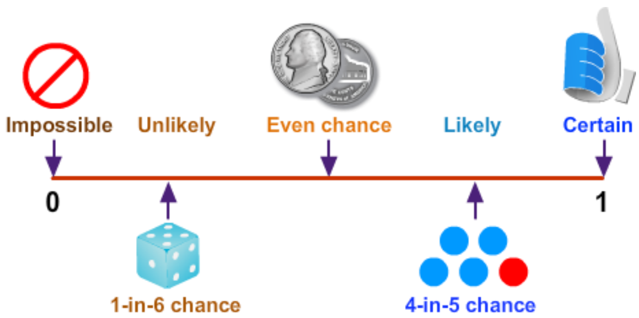
Sophie Kardi skardi@farlingaye.suffolk.sch.uk

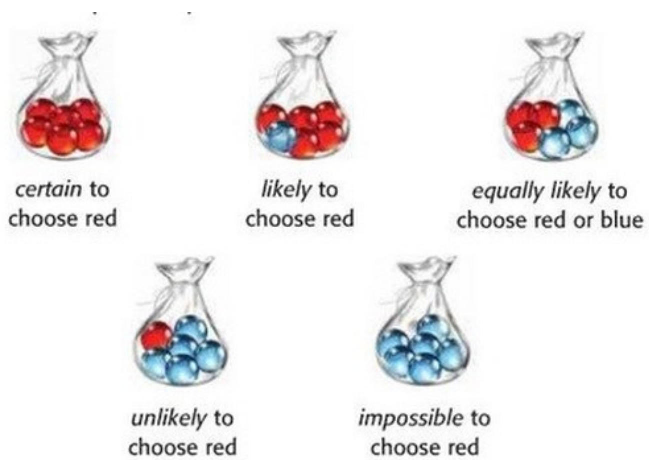
Probability Lines

Websites, Resources & Activities

Probability Cards (next page) - nice to do on desks with whiteboard pens if you have wet wipes at hand

Standards Unit S2 (see next page): more advanced probability statements to help tackle lots of common misconceptions, with excellent explanations/lesson structure included.





Misconceptions

The probability of getting a "n" on a dice is n/6: this comes from not understanding that a probability is a fraction where the numerator indicates how many ways there are to achieve the outcome "n". A sound understanding of listing outcomes helps here.

But miss, a shark could appear in my nostril:

Somehow, the word spontaneously seems to help to resolve this one! As in, "no Daniel, a shark could not **spontaneously** appear in your nostril … "… cue begrudging acknowledgements & acceptance from Year 8.

It must be 50/50 because it could happen, or it could not… the student here is assigning the same probability to all outcomes, even if one is more likely than the other.

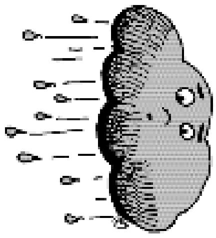
Why this is worth doing

- a starting point to find out what your students know about probability & what misconceptions they have

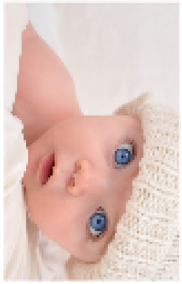
- encourage students to describe probabilities as fractions, percentages and decimals - gives a context and a use to otherwise routine skills

- encourage students to think more about the number of outcomes: get them to write their own statements for a given probability i.e.

    - write me a probability statement where the probability is 2/5
    - write me a statement where the probability is 70%



It will rain tomorrow

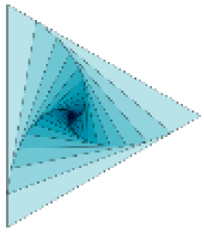


The next baby born is a girl

It will snow tomorrow

A shark suddenly appears in your left nostril

Leicester will win the Premier League



A triangle has three sides

You pick a card from a deck and it is **not** a club

You throw a fair die and get a "2"

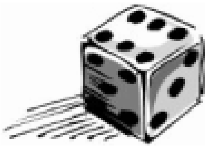
The chance that you've already had your birthday this year

You throw a fair die and get a "1"

S2 Card set A – *True, false or unsure?*

A

When you roll a fair six-sided die, it is harder to roll a six than a four.



B

Scoring a total of three with two dice is twice as likely as scoring a total of two.

C

In a lottery, the six numbers 3, 12, 26, 37, 44, 45 are more likely to come up than the six numbers 1, 2, 3, 4, 5, 6.

D

When two coins are tossed there are three possible outcomes: two heads, one head or no heads. The probability of two heads is therefore $\frac{1}{3}$.

E

There are three outcomes in a football match: win, lose or draw. The probability of winning is therefore $\frac{1}{3}$.

F

In a 'true or false?' quiz with ten questions, you are certain to get five right if you just guess.

G

If you toss a fair coin five times and get five heads in a row, the next time you toss the coin it is more likely to show a tail than a head.

H

In a group of ten learners, the probability of two learners being born on the same day of the week is 1.

I

If a family has already got four boys, then the next baby is more likely to be a girl than a boy.

J

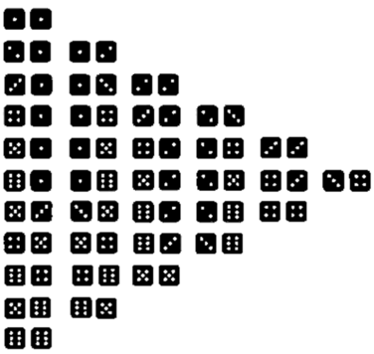
The probability of getting exactly three heads in six coin tosses is $\frac{1}{2}$.

Listing outcomes

How many different ways can these 4 creatures stand in line?

Challenge:
There are a number of methods to do this. How many can you find?

Sample Space Diagrams: there are a number of different ways to list outcomes systematically. Allow students to come up with their own methods. Below (left) is a method that students often use to represent the outcomes on two dice - adequate but students often miss out repeats - for example, they don't include both (1, 3) and (3, 1). Introduce your students to sample space diagrams (below, right) - they minimise the work-load, are adaptable to different events, and ensure *all* outcomes are listed.



	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

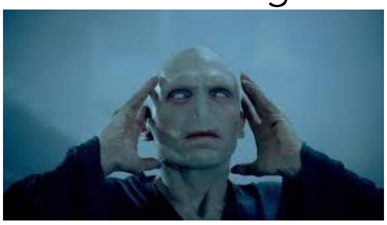
http://donsteward.blogspot.co.uk/2014/01/dibingo.html

Why this is worth doing

- encourages students to systematically organise outcomes as they work - an easy, tangible way to emphasise this key skill
- the total number of outcomes for an event is the denominator in each probability - focusing on listing all outcomes will help students quickly recognise the form their fraction should take.
- opportunity to discuss uses of factorials

http://ed.ted.com/lessons/how-many-ways-can-you-arrange-a-deck-of-cards-yannay-khaikin

star challenge



How many ways were there for Tom Marvolo Riddle to rearrange his name? What was the probability of choosing Lord Voldemort?

Misconceptions

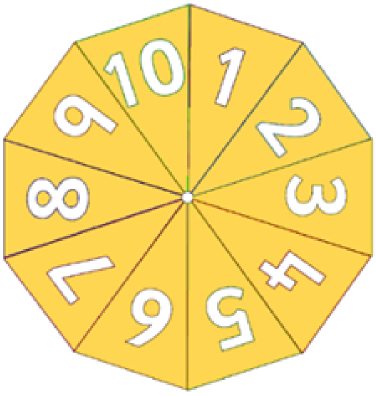
All events are equally likely: e.g. The idea that something has a 50% chance of happening because "it might happen, or it might not". Possible activity: Investigate listing outcomes of events and recognising that some outcomes can occur in more ways than others

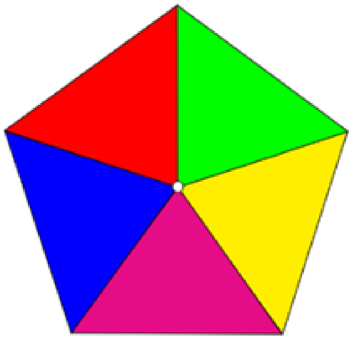
If events are random then the results of a set of independent events are equally likely: e. g. getting two heads is equally as likely as getting a head and a tail. Possible activity: Listing outcomes and discussing how HT and TH are identical but there are two ways for it to occur – so more likely.

Experimental Probability

Experimental Probability is a probability calculated during experiments, direct observation, experience or practice. *Relative Frequency* is another name for the experimental probability.

Students often don't understand the term "relative frequency" - it is worth spending some time making sure they are comfortable using this vocabulary.





Drawing Pin Experiments

https://www.teachitmaths.co.uk/attachments/ 16290/drawing-pin-probability-experiment.pdf

Create biased spinners and explore through experimental probability.

Investigate the Buttered Toast Phenomenon

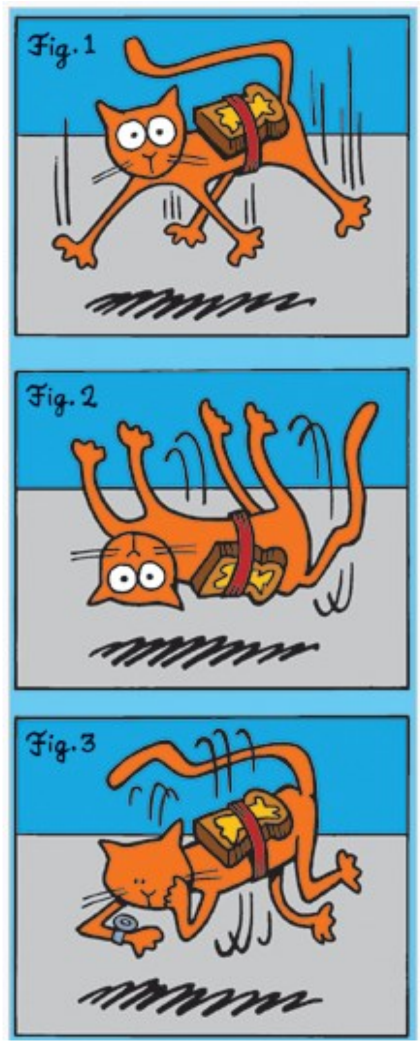
I never had a slice of bread,
Particularly large and wide,
That did not fall upon the floor,
And always on the buttered side!

James Payn, 1884

Misconceptions

If you are working out an experimental probability, the size of the sample doesn't matter E.g. you flip a coin twice and get two heads so decide it is biased, versus flipping a coin 100 times and getting 80 heads and saying it is biased. <u>Possible activity:</u> debate – how many experiments does it take for something to become biased? Encourage students to play devil's advocate.

"Lucky/Unlucky" numbers can influence random events E.g. less likely to get a 6 when you roll a die because it's "lucky". <u>Possible activity:</u> experiment to find the relative frequency of rolling a 6. Create tally chart/frequency chart – opportunity to then extend this into bar charts, pie charts etc to consolidate understanding.



The Buttered Cat Phenomenon

Theoretical Probability

Theoretical Probability is based on your logical reasoning about an event., and is the likeliness of an event occurring based on all of the possible outcomes. There are two key rules to understand - the "And" rule and the "Or" rule.

The And Rule: P(A and B) = P(A) x P(B)

For events with successive outcomes, e.g. *this* happens and then *that* happens.

Great opportunities to multiply fractions and decimals

The Or Rule: P(A or B) = P(A) + P(B)

For combining different (mutually exclusive) outcomes e.g. *this* or *that* happens.

Great opportunity for adding & converting fractions, decimals and percentages.

	1	2	3	4	5	6
Head	h1	h2	h3	h4	h5	h6
Tail	t1	t2	t3	t4	t5	t6

P(head **and** 3) = P(head) x P(3)
= 1/2 x 1/6
= 1/12

(one out of 12 outcomes in diagram)

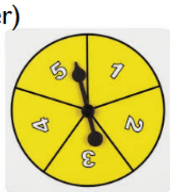
	1	2	3	4	5	6
Head	h1	h2	h3	h4	h5	h6
Tail	t1	t2	t3	t4	t5	t6

P(1 **or** 3 **or** 5) = P(1) + P(3) + P(5)
= 2/12 + 2/12 + 2/12
= 6/12 = 1/2

(6 out of 12 outcomes in diagram)

Find the following probabilities - leave your answers as fractions:

a) Rolling a 5 followed by a 6 on a die
b) Rolling an even number followed by a 5 on a die
c) Rolling a prime number on a die followed by flipping a coin and getting a head
d) Flipping a coin and getting a head and followed by a tail
e) Flipping a coin and getting a head and a tail (in any order)
f) Getting a 3 on this spinner and then getting a square number on a die

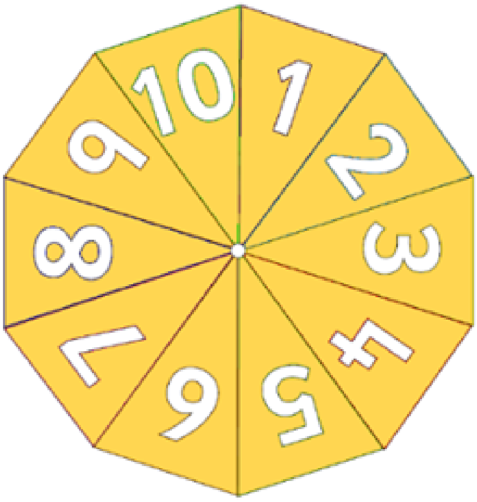


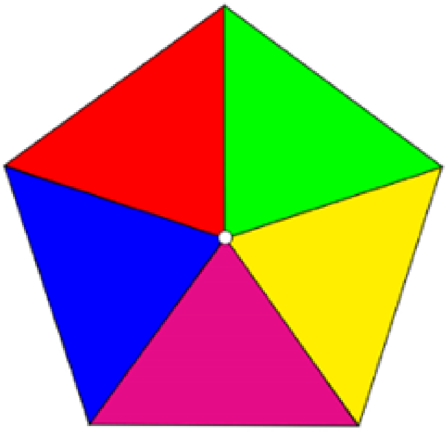
g) Rolling a prime number on a 20-sided dice, and then flipping a coin and getting a head
h) Flipping a coin and getting (in <u>this</u> order) 3 heads, a tail, and another head
i) Flipping a coin and getting (in any order) 3 heads, a tail, and another head
j) Getting a prime number on the spinner, followed by a square number, followed by an odd number

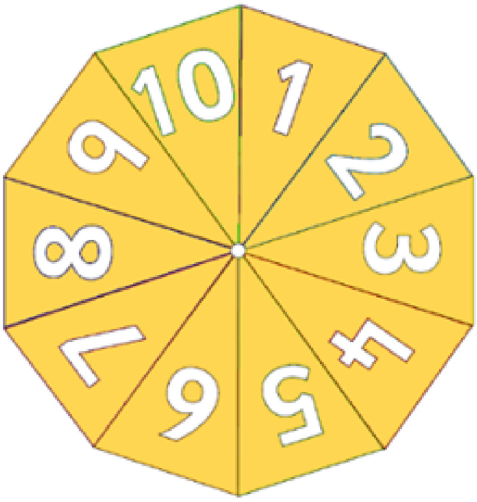
***** Create some questions of your own, with solutions ******

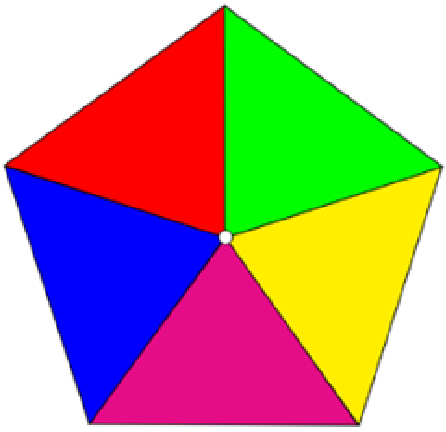
A quick set of questions for practising **and/or** rules.
This ideally follows work on listing outcomes.

It works particularly well if you use sample space diagrams to tackle any misconceptions students have about the **and/or** rules.









probability with words

http://donsteward.blogspot.co.uk/2013/09/probability-and-words.html

(1) S N E E Z E

what is the probability of picking an 'E' out of the letters in SNEEZE ?

(2) give some words where the probability of picking a letter 'a' from a word is ⅓
try to find words with different lengths

(3) give some words where the probability of picking a letter 'e' from a word is ⅓
try to find words with different lengths

(4) give some words where the probability of picking a vowel from a word is greater than ½
try to find words with different lengths

(5) give some words where the probability of picking a consonant from a word is ¾
try to find words with different lengths

(6) give some words where the probability of picking a consonant from a word is close to 1
how close can you get?

(7) put these words in order for the probability of picking a 'T' from them:
ATTEMPTERS
DAUNT
ENTENTES

(8) what do these words have in common (in terms of probabilities)?
BETTER
TEA
BEVERAGES

(9) what do these words have in common (in terms of probabilities)?
ADDITION
SIDE
DODECAHEDRON

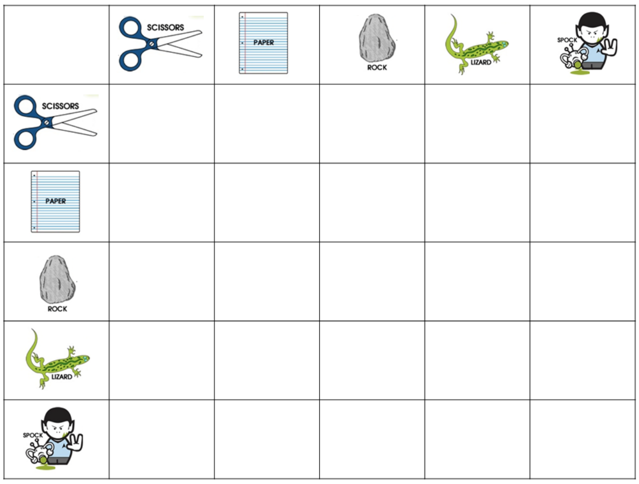
(10) what do these words have in common (in terms of probabilities)?
DAMAGE
READ
UNIDENTIFIED
SOLITUDE

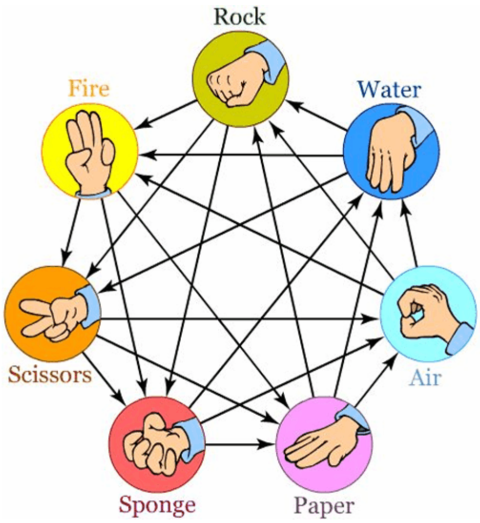
Rock, Paper, Scissors ...

Andrew Gemmell: http://farlingayemaths.blogspot.co.uk/search/label/rock%20paper%20scissors

rock, paper, scissors, lizard, spock

is the game fair or unfair?





Can you find a rule for P(draw), P(Win), or P(Lose)?
Can you write this as a formula (using symbols/algebra?)

RPS-N	Number of Outcomes	P(Draw)	P(Win)	P(Lose)
3				
5				
7				
9				
15				
...				
n				

These resources allow students to quickly understand and grasp theoretical probability and identify patterns. Drawing sample space diagrams becomes time consuming for games such as RPS-11 and RPS-15, and so they often opt to find patterns instead! Because of this, its relatively easy to work with a class to obtain a word formula/ algebraic rules for the probabilities of winning, losing and drawing.

Investigations

Websites, Resources & Activities

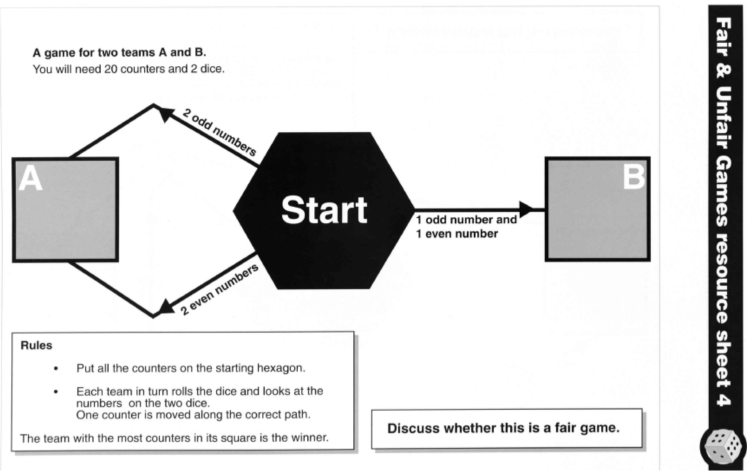
Ten Ticks: Fair or Unfair Games, Level 4 Pack 6 (pages 31- 36)

Ten Ticks: More Fair or Unfair games. Level 6, Pack 6

Don Steward - Quirky Dice (next page) http://don) steward.blogspot.co.uk/2014/03/quirky-dice_3699.html

CIMT Probability: http://www.cimt.org.uk/resources/help/h10prob2.pdf

Excellent discussion of misconceptions and resources to tackle them.



Smile: Fair or Unfair Games. Set of 6 games that require an understanding of sample spaces and the **and/or** rules to analyse. Great for comparing experimental to theoretical results.

Includes solutions and suggested lesson plan with extension activities.

George Sicherman's pair of dice: sample space

Don Steward
http://donsteward. blogspot.co.uk/
2014/03/sicherman-dice.html

Lovely activity to encourage students to think what makes a "normal" pair of dice. Create them and encourage students to run an experiment where they add the results of the dice when thrown. Then investigate through sample spaces.

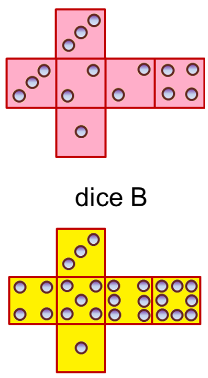
dice A

dice B

+	1	2	2	3	3	4
1	2	3	3	4	4	5
3	4	5	5	6	6	7
4	5	6	6	7	7	8
5	6	7	7	8	8	9
6	7	8	8	9	9	10
8	9	10	10	11	11	12

dice A

dice B



Misconceptions

With spinners, probability is determined by number of sections rather than size of angles in the sections E.g. a spinner is split into three sections - one half and two quarters - so the probability of each outcome is one third. <u>Possible activity:</u> experiment to find the relative frequency of each section. Compare to their expected probabilities – do they match? A great opportunity to practice finding fractions of a circle by using angles, and to practice comparing fractions.

with Probability

QUIRKY DICE

quirky dice (1)

complete the sample space tables to see which dice would have an advantage if two dice were used in a 'highest score wins' competition

dice A

	9	
2	4	9
	2	
	4	

dice B

	6	
1	8	6
	1	
	8	

dice C

	7	
3	5	7
	3	
	5	

which dice is best of all three?

dice A

dice B	2	2	4	4	9	9
1						
1						
6						
6						
8						
8						

dice …. beats dice …. in the long run

dice C

dice B	3	3	5	5	7	7
1						
1						
6						
6						
8						
8						

dice …. beats dice …. in the long run

dice A

dice C	2	2	4	4	9	9
3						
3						
5						
5						
7						
7						

dice …. beats dice …. in the long run

quirky dice (2)

complete the sample space tables to see which dice would have an advantage if two dice were used in a 'highest score wins' competition

dice A

	6	
2	2	6
	2	
	2	

dice B

	5	
1	5	5
	5	
	1	

dice C

	4	
3	4	4
	3	
	3	

which dice is best of all three?

dice A

dice B	2	2	2	2	6	6
1						
1						
5						
5						
5						
5						

dice …. beats dice …. in the long run

dice C

dice B	3	3	3	4	4	4
1						
1						
5						
5						
5						
5						

dice …. beats dice …. in the long run

dice A

dice C	2	2	2	2	6	6
3						
3						
3						
4						
4						
4						

dice …. beats dice …. in the long run

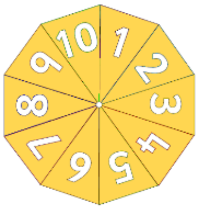
Activity exploring the unusual nature of non-transitive dice. Great as an investigation & reinforces skills students have learnt. The game takes a similar structure to Rock Paper Scissors in that every dice can be beaten by another. Ideas for how to structure a lesson/probing questions to ask students at http://nrich.maths.org/7541.

Extension task at http://nrich.maths.org/623 - four dice to consider with

Probability Trees

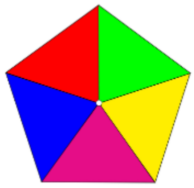
A tree diagram is simply a way of representing a sequence of events. Tree diagrams are particularly useful in probability since they record all possible outcomes in a clear and uncomplicated manner.

https://nrich.maths.org/7288 for a detailed introduction into tree diagrams

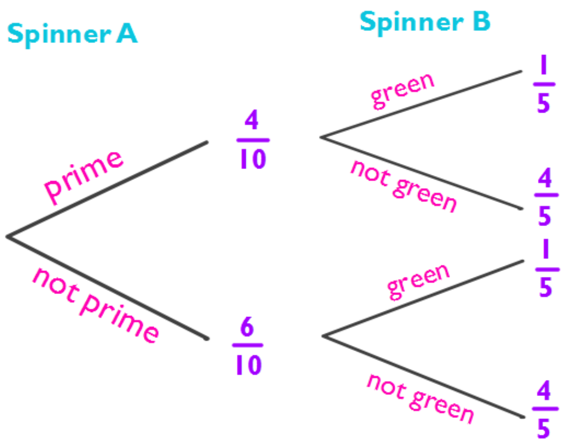


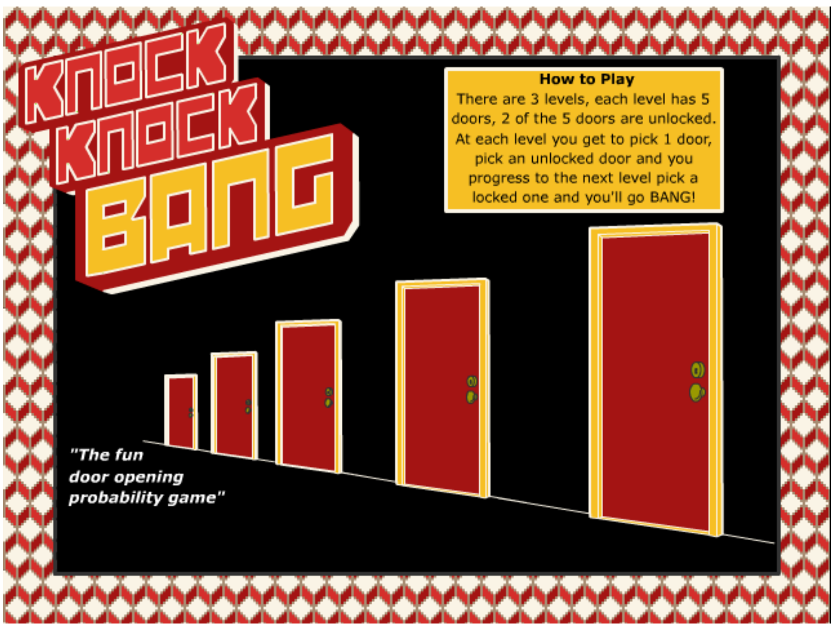
Spinner A

There are lots of situations that we can look at using tree diagrams.



Spinner B





Extension Task

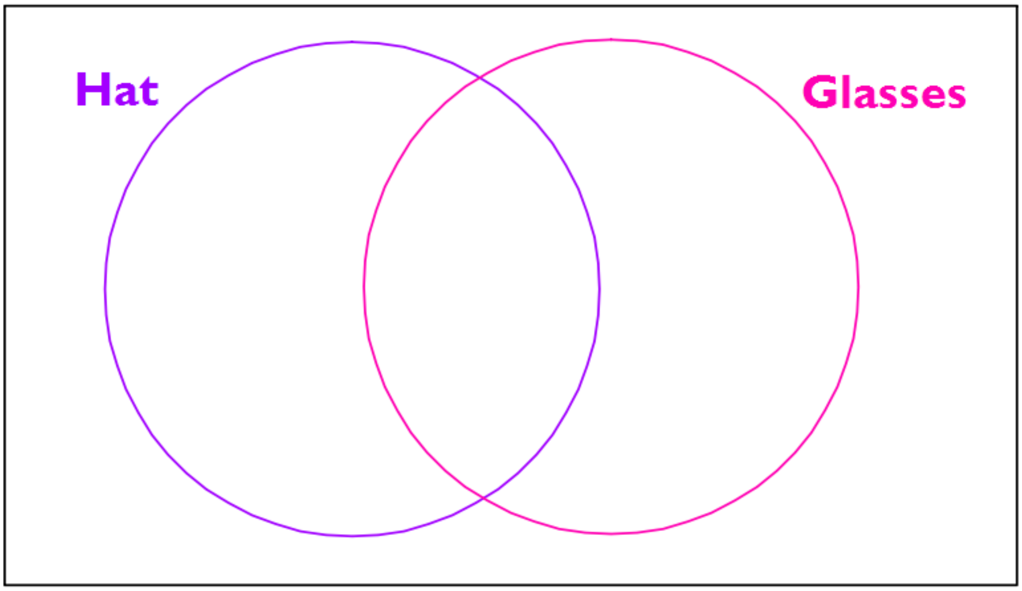
www.tes.com/teaching-resource/knock-knock-bang-tree-diagrams-6301897

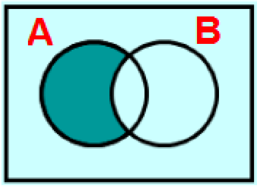
Online Simulator: www.censusatschool.com/questionnaires/game/index.html

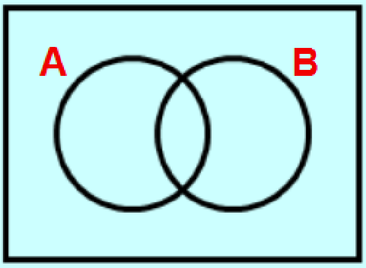
Venn Diagrams

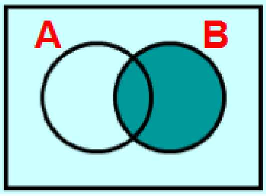
Venn Diagrams have been introduced into the 9-1 GCSE Maths. They are another way to represent the relationship between two events, and allow students to access many more complicated ideas such as conditional probability and set notation.

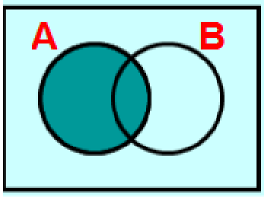
The following resources are from **Maddy Chapman:** mchapman@farlingaye.suffolk.sch.uk









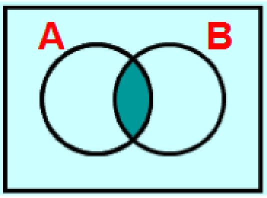


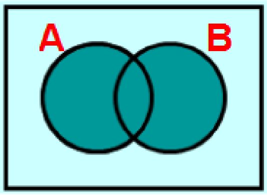
P(A) P(B) P(A')

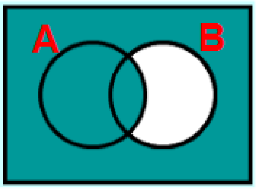
$P(A \cap B)$ $P(A \cup B)$

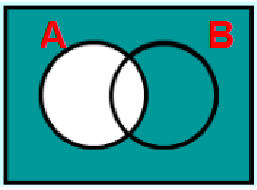
$P(A' \cap B')$ $P(A \cap B')$

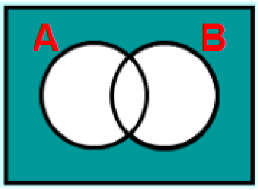
$P(A \cup B')$







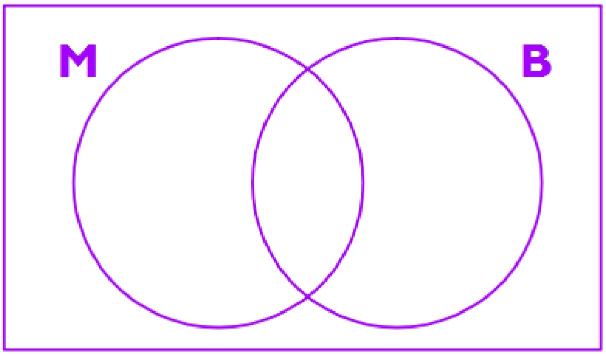




In an group of 100 students:

50 study Mathematics, 29 study Biology, 13 study both.

Draw a Venn Diagram and use it to find:



$P(M) =$

$P(B) =$

$P(M \cup B) =$

$P(M \cap B) =$

$P(M') =$

$P(B') =$

$P(M \cap B') =$

Notes Space