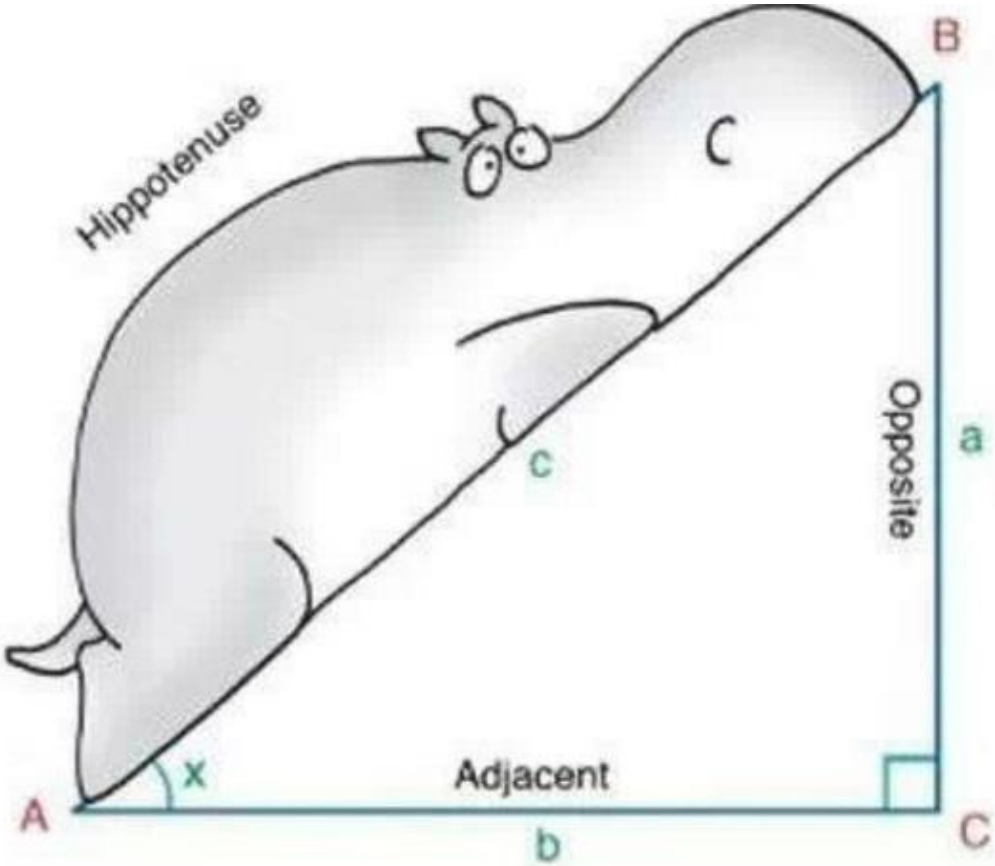
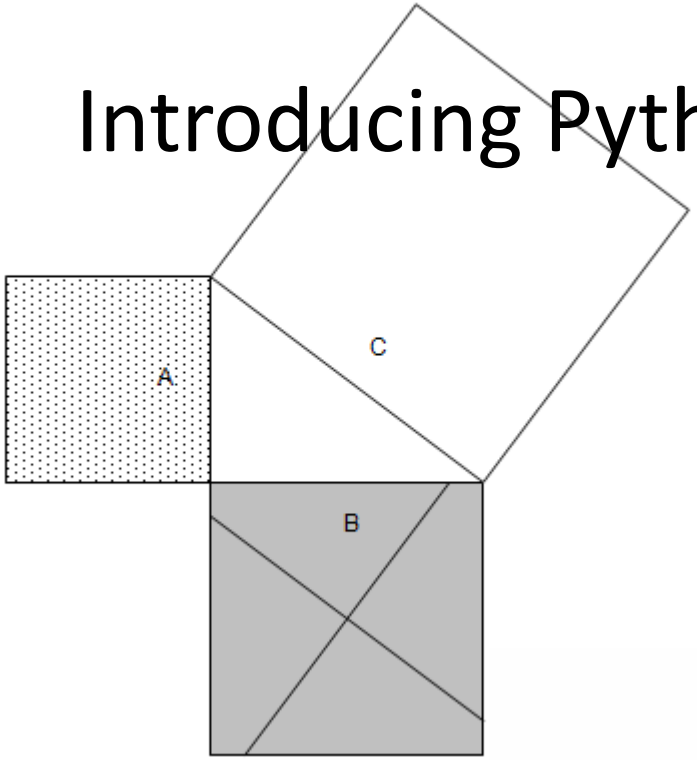
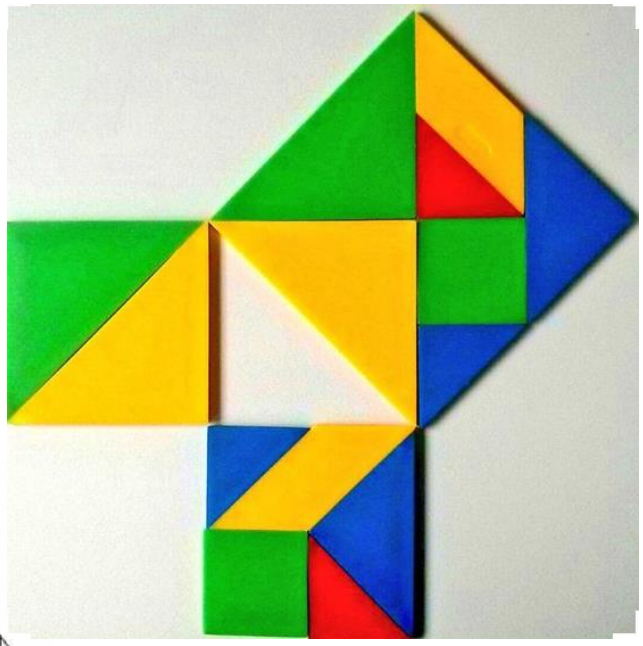
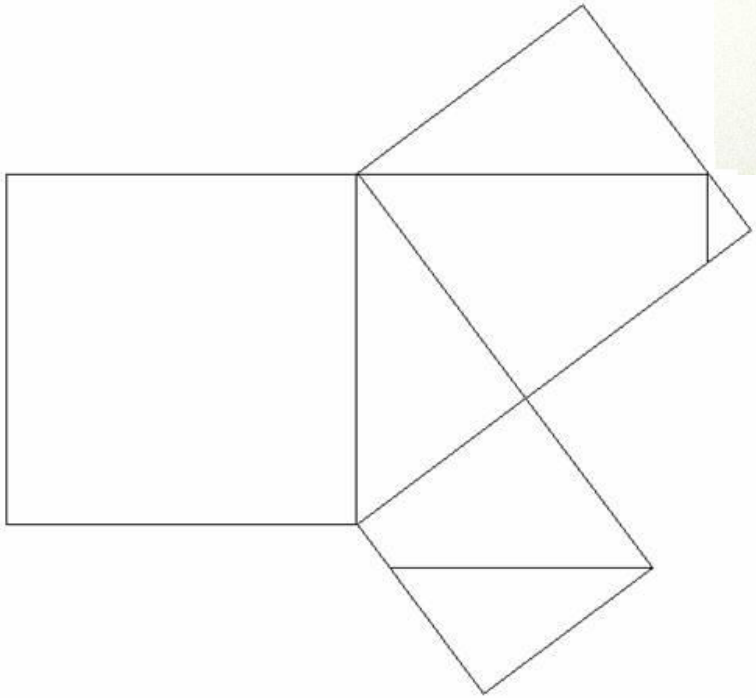
Pythagoras and Trigonometry



Introducing Pythagoras using CPA







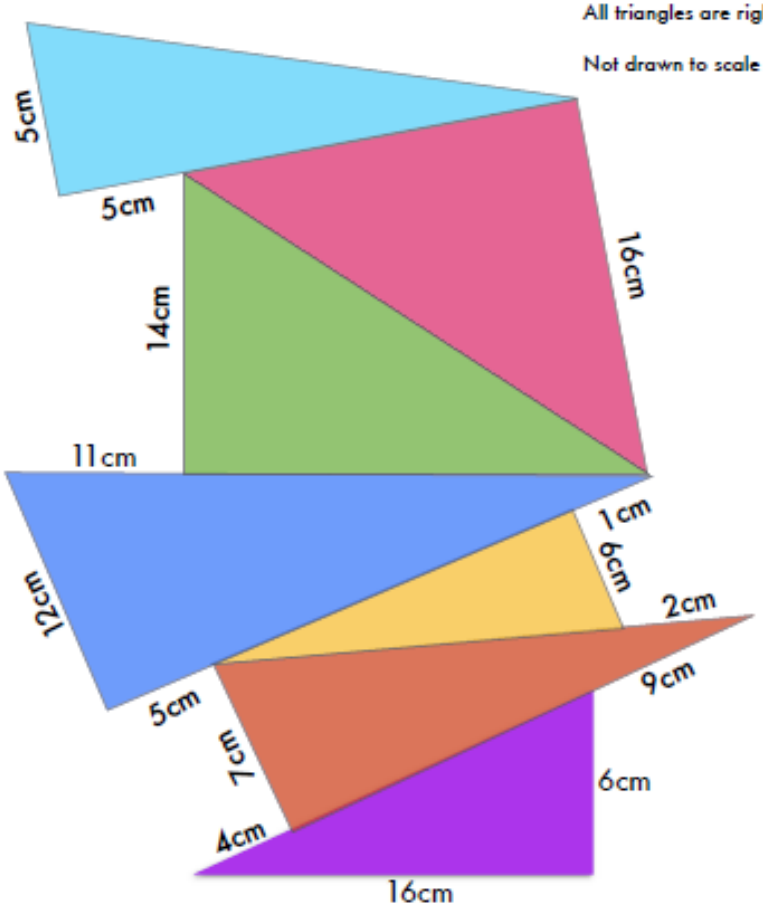
Pythagoras Pile Up

Race to the top?

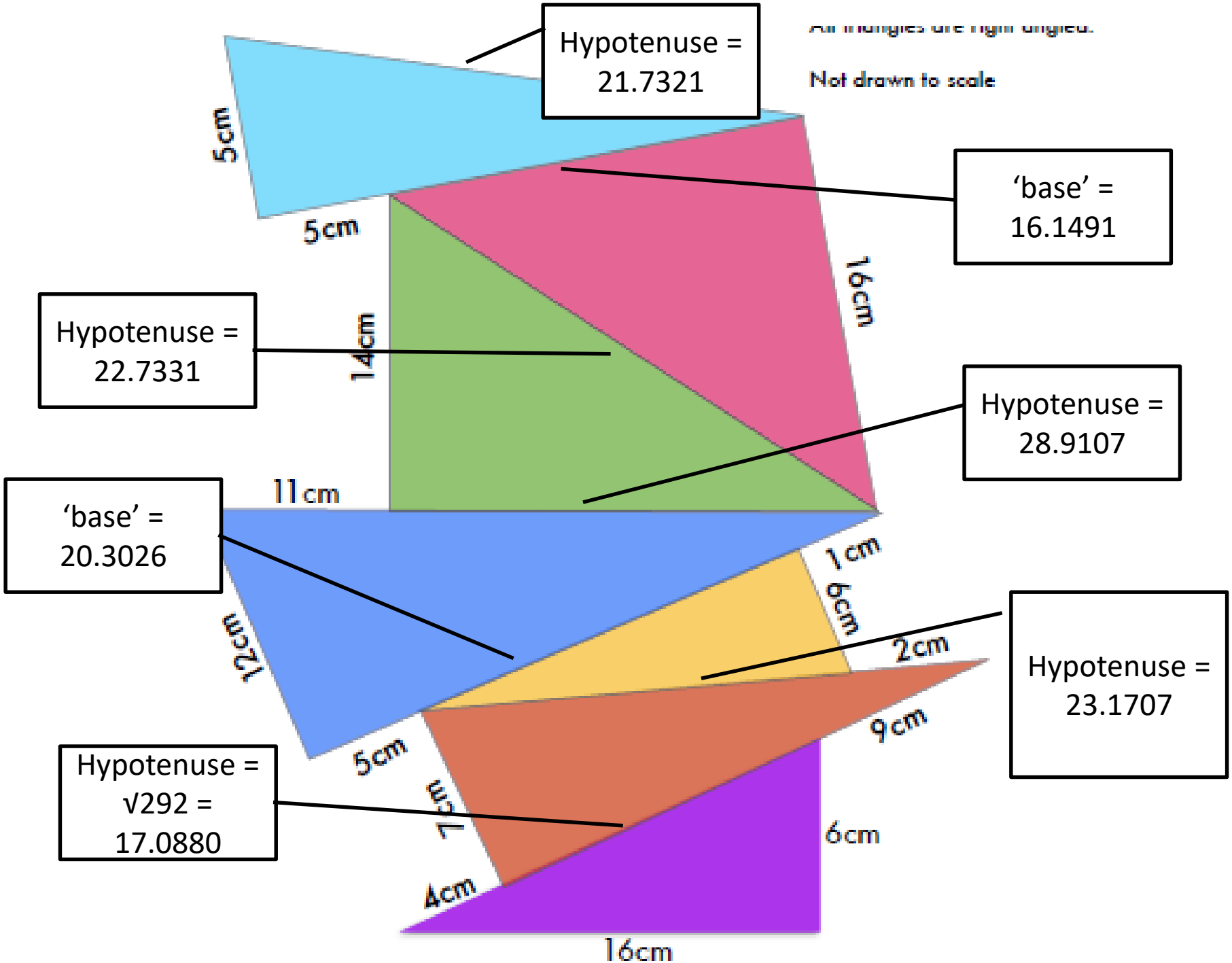
Find the hypotenuse of the top triangle.

All triangles are right angled.

Not drawn to scale



Based on a concept from greatmathsteachingideas.com and a design by @mrallanmaths


All triangles are right angled.
Not drawn to scale
Hypotenuse = 21.7321
5cm
5cm
'base' = 16.1491
16cm
Hypotenuse = 22.7331
14cm
Hypotenuse = 28.9107
11cm
'base' = 20.3026
1cm
9cm
12cm
2cm
Hypotenuse = 23.1707
9cm
5cm
Hypotenuse = √292 = 17.0880
7cm
6cm
4cm
16cm

What Prior knowledge is needed?

- Understanding of the concept of area
- Use squares and square roots
- Know how to solve for a variable by "undoing"
- Use variables to represent lengths.
- Properties of a right triangle.
- Understand order of operations when solving an equation
- Plot ordered pairs on a coordinate grid.
- Classify angles and triangles as acute, right, and obtuse.

Maths 4 Real **Net NotesPLUS**

Programme 6 Worksheet 1: Programme Questions

Answer the following questions while you watch the programme.

1. What is the longest side of a right-angled triangle called?

2. Write down the formula for Pythagoras' Theorem given in the programme.

3. What is the height of the tower to which the

7. Write down another set of three numbers that form a Pythagorean triple.

8. In 'Ti
boat i

9. What
Trash

Maths 4 Real **Net NotesPLUS**

Programme 6 Worksheet 2: Tick or Trash

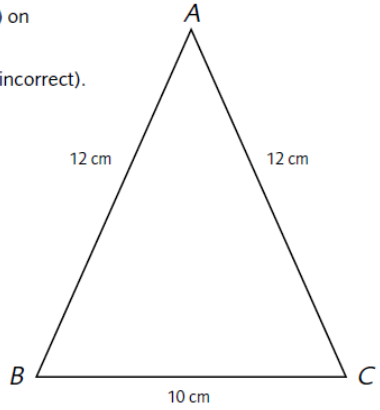
Here are some questions and answers (by Students A and B) on Pythagoras' Theorem.
Decide which answers to Tick (correct) and which to Trash (incorrect).
Give reasons.

Question 1

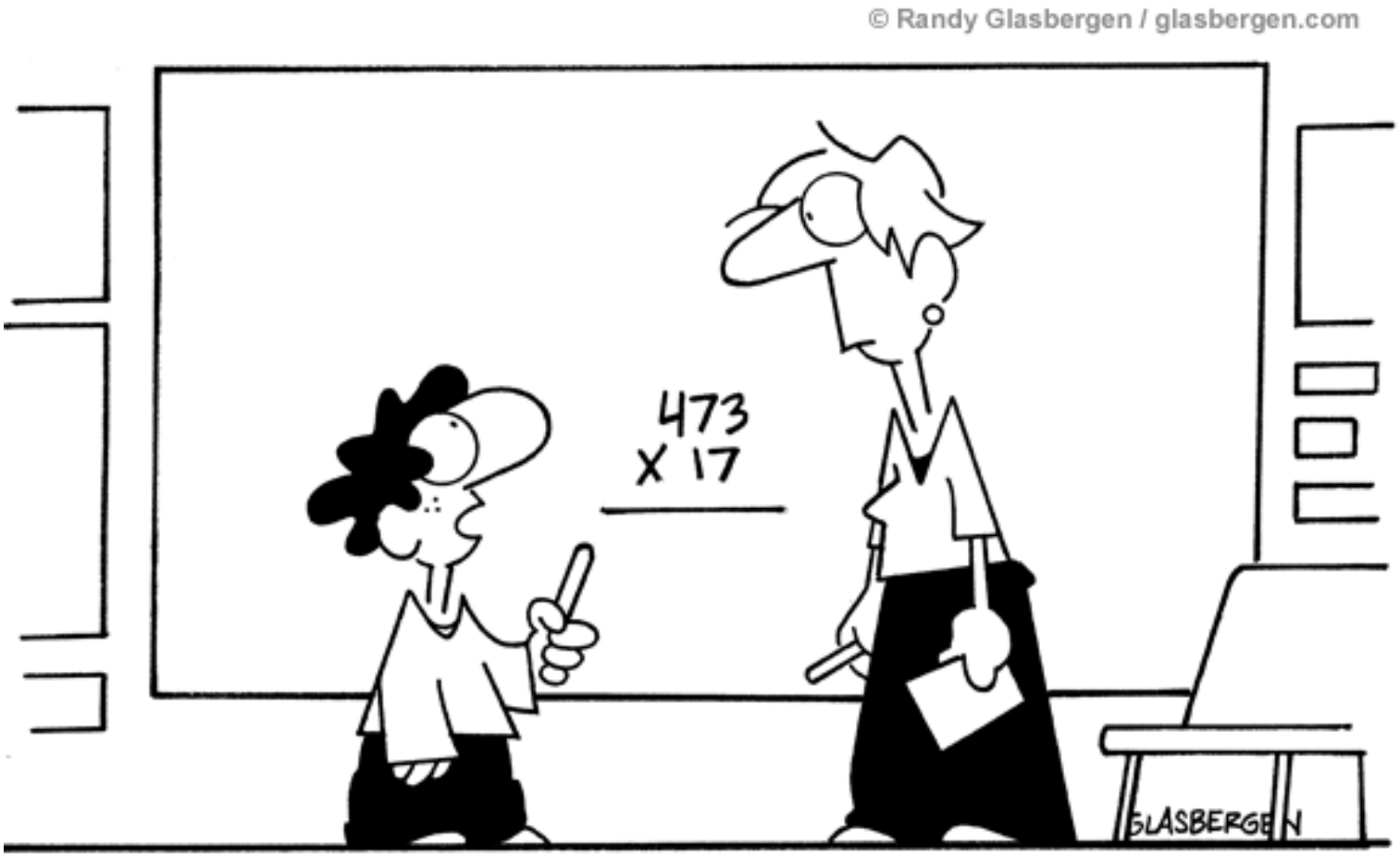
ABC is an isosceles triangle.

$AB = AC = 12\text{cm}$
$BC = 10\text{cm}$

Calculate the perpendicular distance from *A* to *BC*.



The Mistake Game



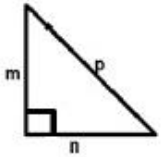
"If we learn from our mistakes, shouldn't I make as many mistakes as possible?"

At the end of the activity

- When do you add and when do you subtract?
- Where there is more than one right-angled triangle, how do you decide which one to work on first?
- Why is it useful to draw your own diagram?
- In which questions did you have to do a calculation before you could use Pythagoras' Theorem?
- In which questions did you have to 'think backwards' to solve the problem?
- What do you have to do if there do not seem to be any right-angled triangles in the diagram?

THE THEOREM OF PYTHAGORAS

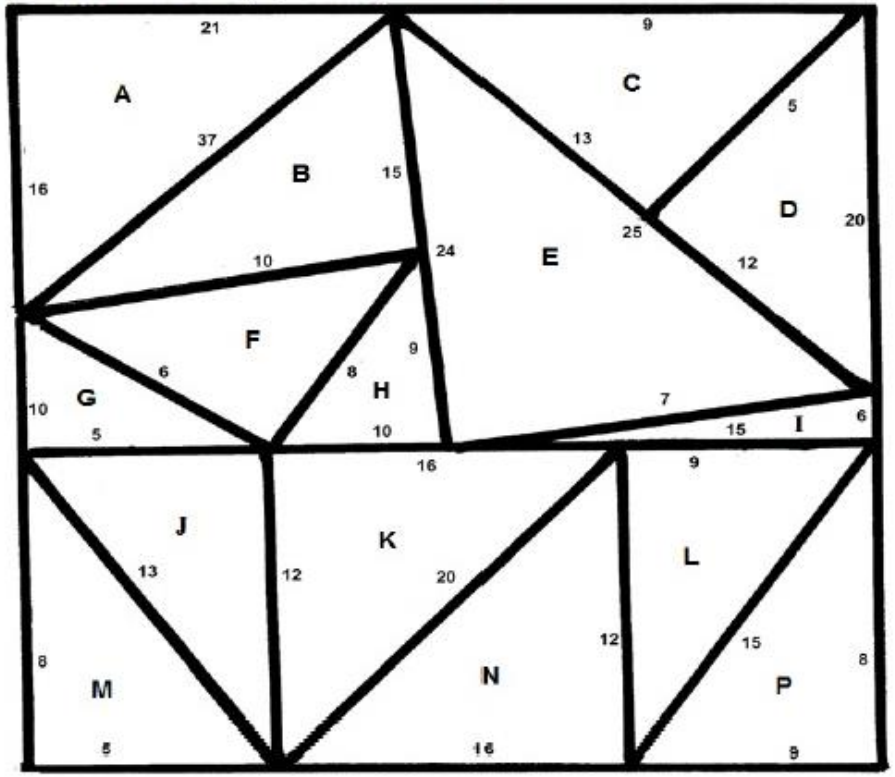
The theorem of Pythagoras says that in any right angled triangle the hypotenuse squared is equal to the sum of the squares of the other 2 sides. For example in the triangle on the right:
$m^2 + n^2 = p^2$.



Using the above information, decide which of the triangles in the shape below are right angled triangles and which are not. Shade them as follows:

Right-angled triangles: red **Non right-angled triangles: blue**

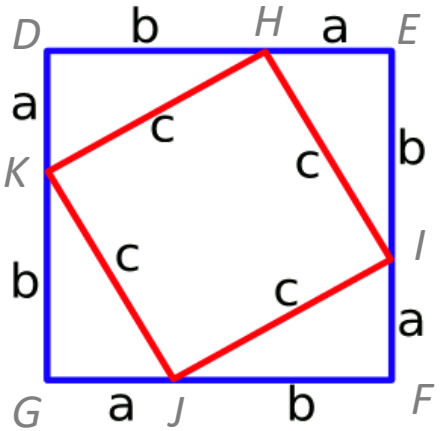
You must hand in another page with all your working out.



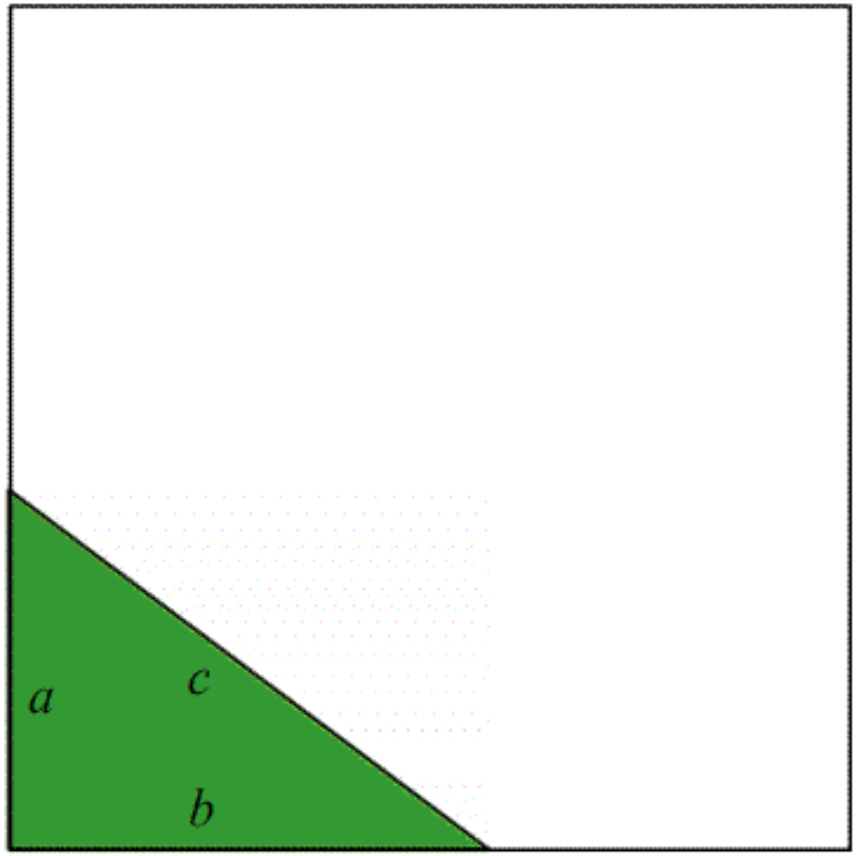
Misconceptions

- ? Works for any triangle
- ? Add or take away
- ? Forget to square root

An Algebraic Proof of Pythagoras' Theorem.



1. Start with any right angled triangle, with sides a and b and hypotenuse of length c. We want to find c in terms of a and b.
2. Make four copies of the triangle and arrange them at the corners of a square as in the diagram.
3. Label all the vertices as shown in the diagram.
4. Explain why the space in the middle of the four triangles (i.e. HIJK) is a square. (all sides equal? All angles 90°? How do you know?)
5. What is the side length of the large square, DEFG, in terms of a and b?
6. Based on your answer to part 5, write an expression for the area of square DEFG in terms of a and b;
   i. In factorised form
   ii. In expanded form
7. What is the area of each of the triangles in terms of a and b?
8. The area of the large square DEFG can be written as the sum of the area of the small square HIJK plus the areas of the four triangles. Write this area in terms of a, b and c.
9. In parts 6 and 8, you have found two different expressions for the area of the large square DEFG. These two expressions for the area must be equal. Put your expressions from 6ii and 8 equal to each other. Simplify the equation as much as possible.
10. Explain how what you have found proves Pythagoras' theorem.



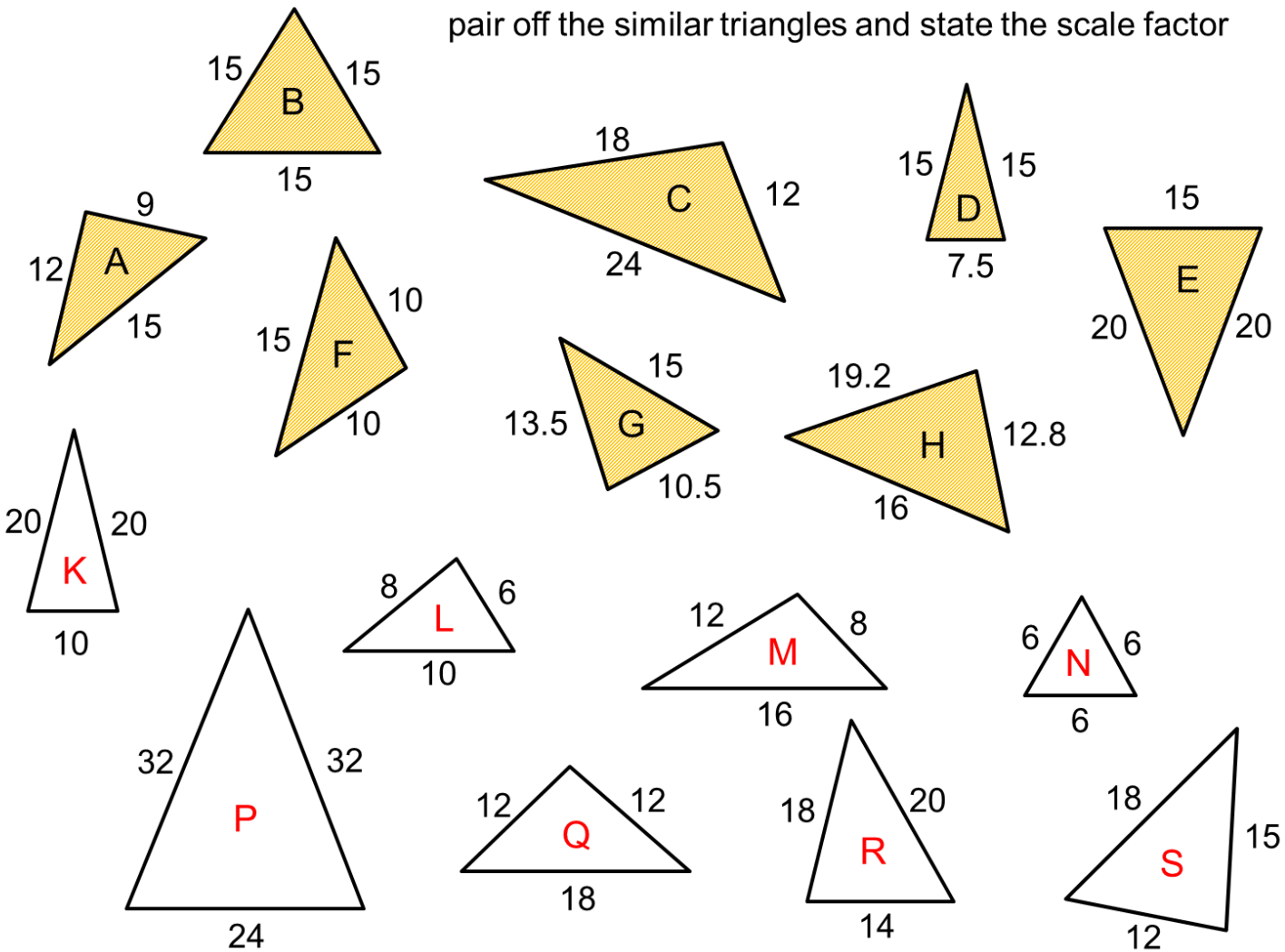
A right triangle, with legs a and b and hypotenuse c.

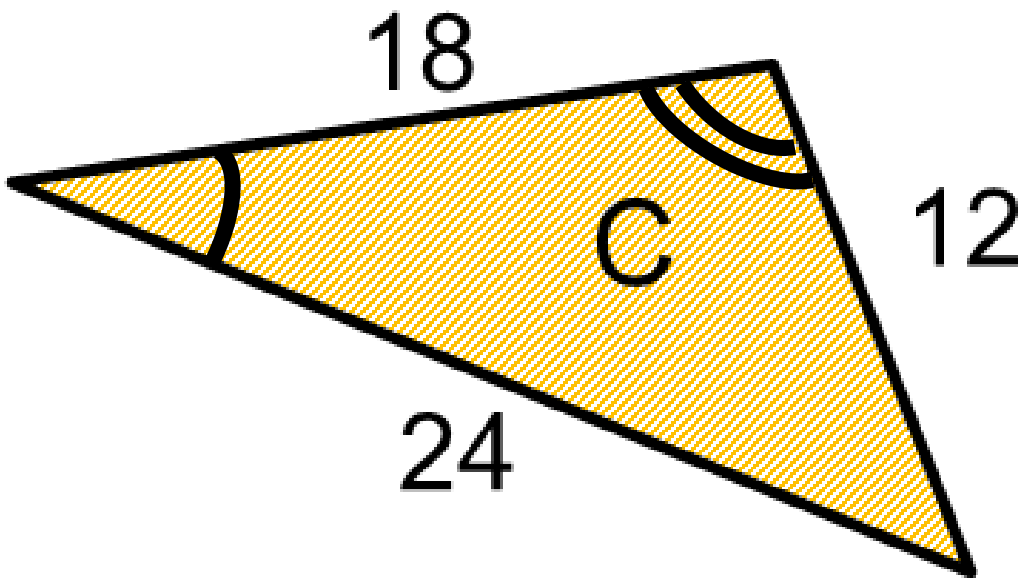
Trigonometry

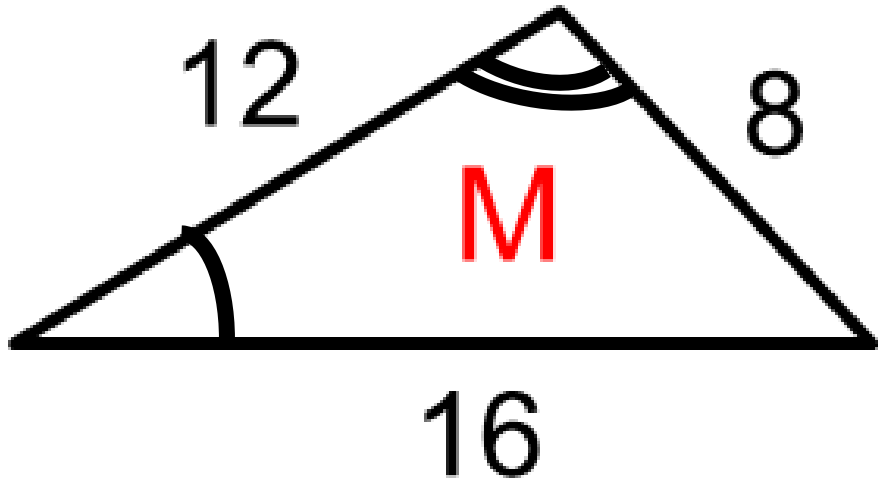
Similar Triangles

angles all the same, sides in the same ratio

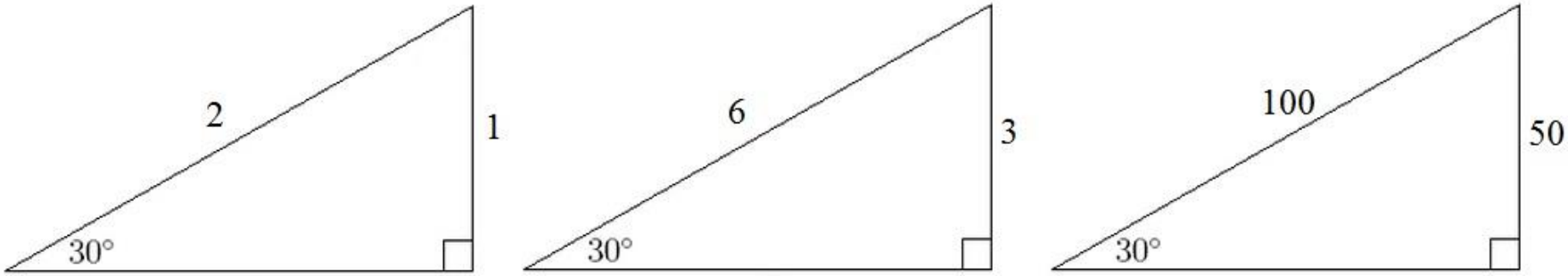
pair off the similar triangles and state the scale factor




18
12
C
24

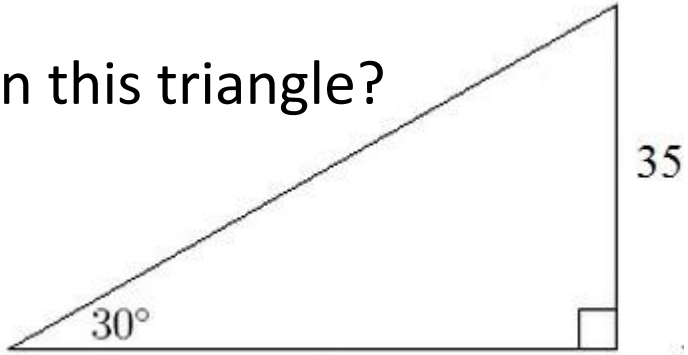

12
8
M
16

What do these triangles have in common?

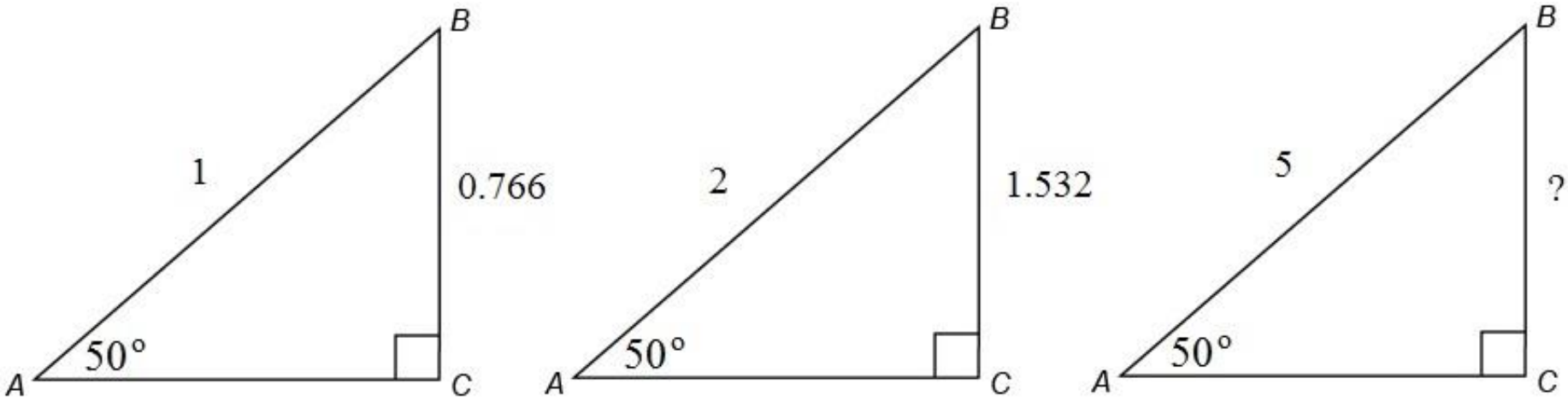


(not drawn to scale)

What is the length of the hypotenuse on this triangle?



And what do these triangles have in common?



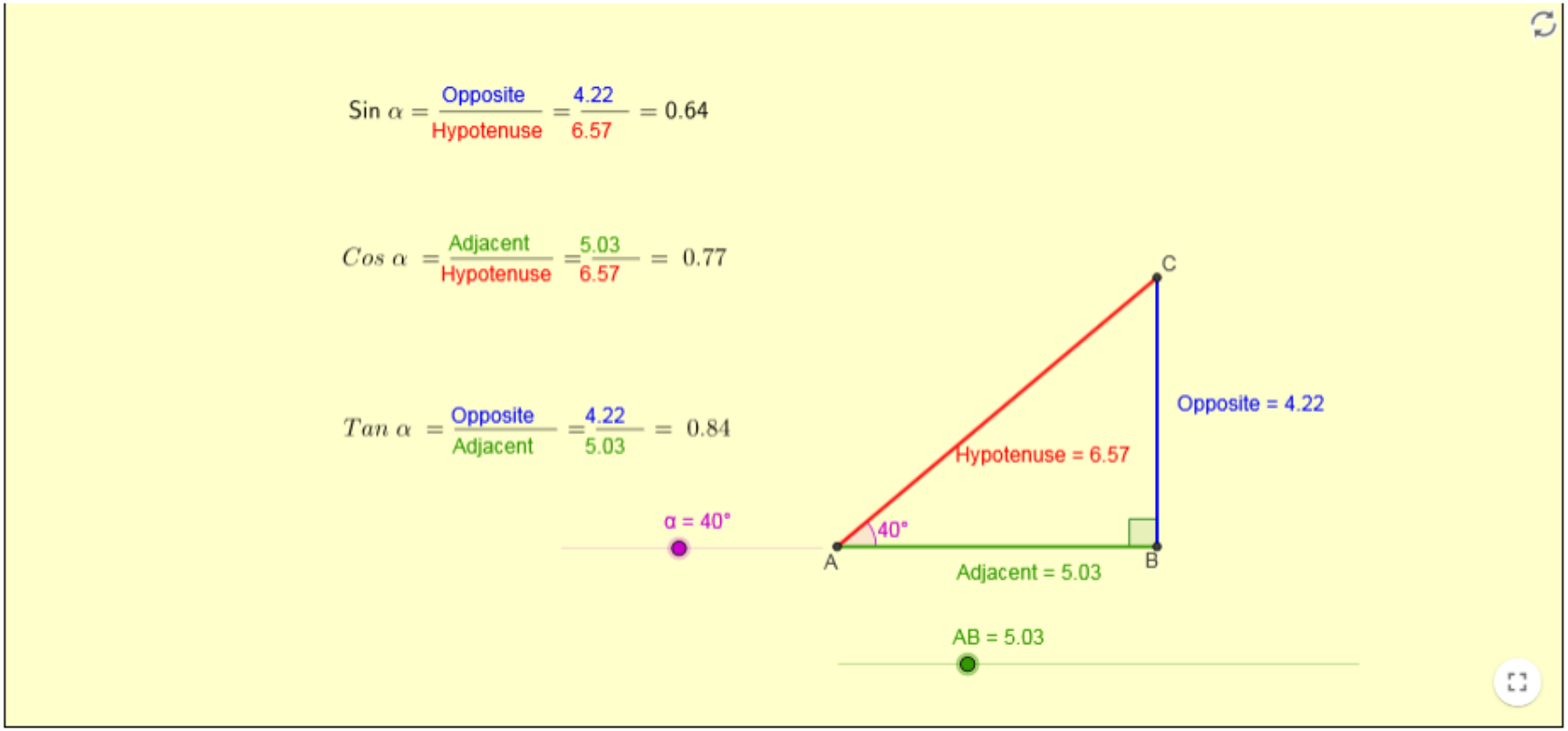
What is the length missing on the third triangle?

Table of Trigonometric Ratios

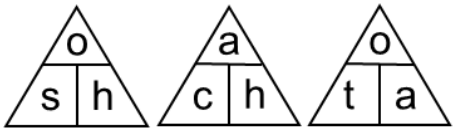
Degrees	Sine (sin)	Cosine (cos)	Tangent (tan)		Degrees	Sine (sin)	Cosine (cos)	Tangent (tan)
1	0.0175	0.9998	0.0175		46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349		47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524		48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699		49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875		50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051		51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228		52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405		53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584		54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763		55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944		56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126		57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309		58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493		59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679		**60**	**0.8660**	**0.5000**	**1.7321**
16	0.2756	0.9613	0.2867		61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057		62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249		63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443		64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640		65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839		66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040		67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245		68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452		69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663		70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877		71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095		72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317		73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543		74	0.9613	0.2756	3.4874
30	**0.5000**	**0.8660**	**0.5774**		75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009		76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249		77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494		78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745		79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002		80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265		81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536		82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813		83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098		84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391		85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693		86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004		87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325		88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657		89	0.9998	0.0175	57.2900
45	**0.7071**	**0.7071**	**1.0000**		90	1.0000	0.0000	

VS.

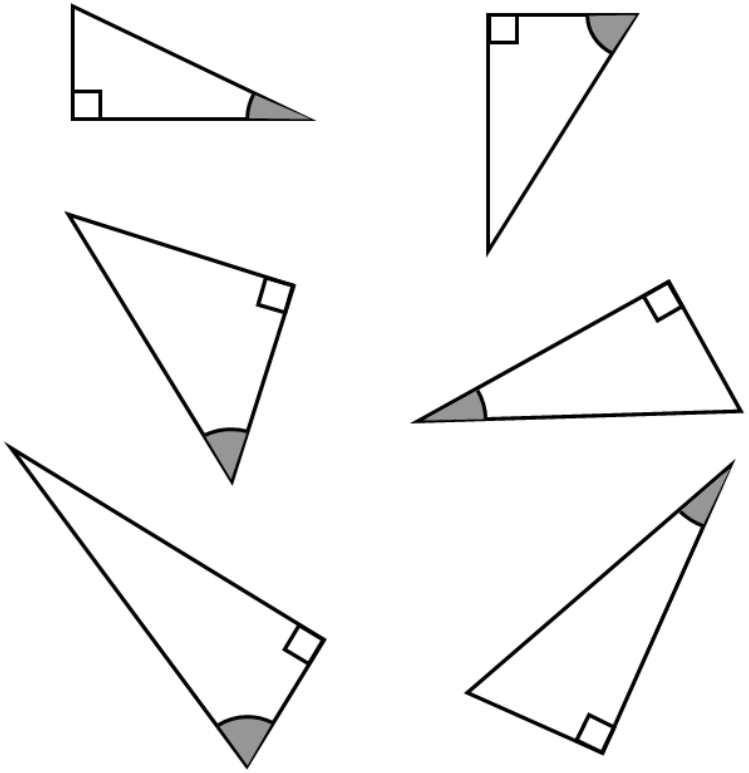
Geogebra


Sin α = Opposite / Hypotenuse = 4.22 / 6.57 = 0.64
Cos α = Adjacent / Hypotenuse = 5.03 / 6.57 = 0.77
Tan α = Opposite / Adjacent = 4.22 / 5.03 = 0.84
C
Opposite = 4.22
Hypotenuse = 6.57
α = 40°
40°
A
B
Adjacent = 5.03
AB = 5.03

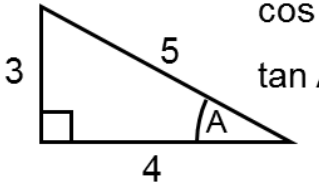
trigonometry



label the sides for the given angle:
o for opposite
a for adjacent (next to)
h for hypotenuse (the longest side)



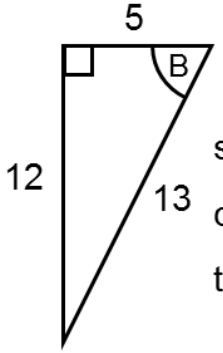
write the three ratios as fractions



sin A =

cos A =

tan A =



sin B =

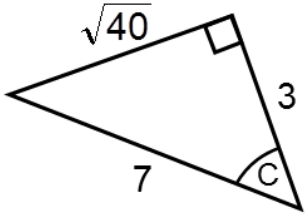
cos B =

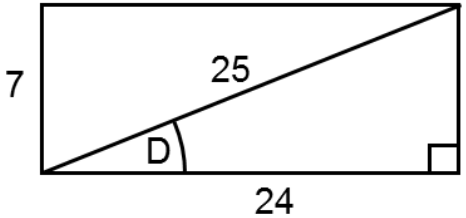
tan B =

sin C =

cos C =

tan C =

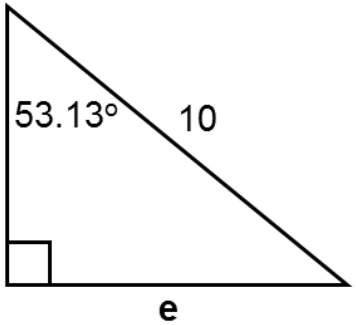




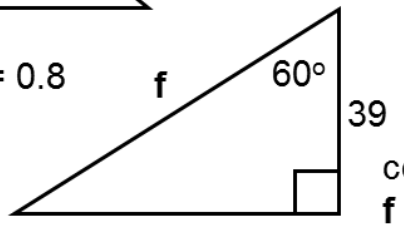
sin D =

cos D =

tan D =



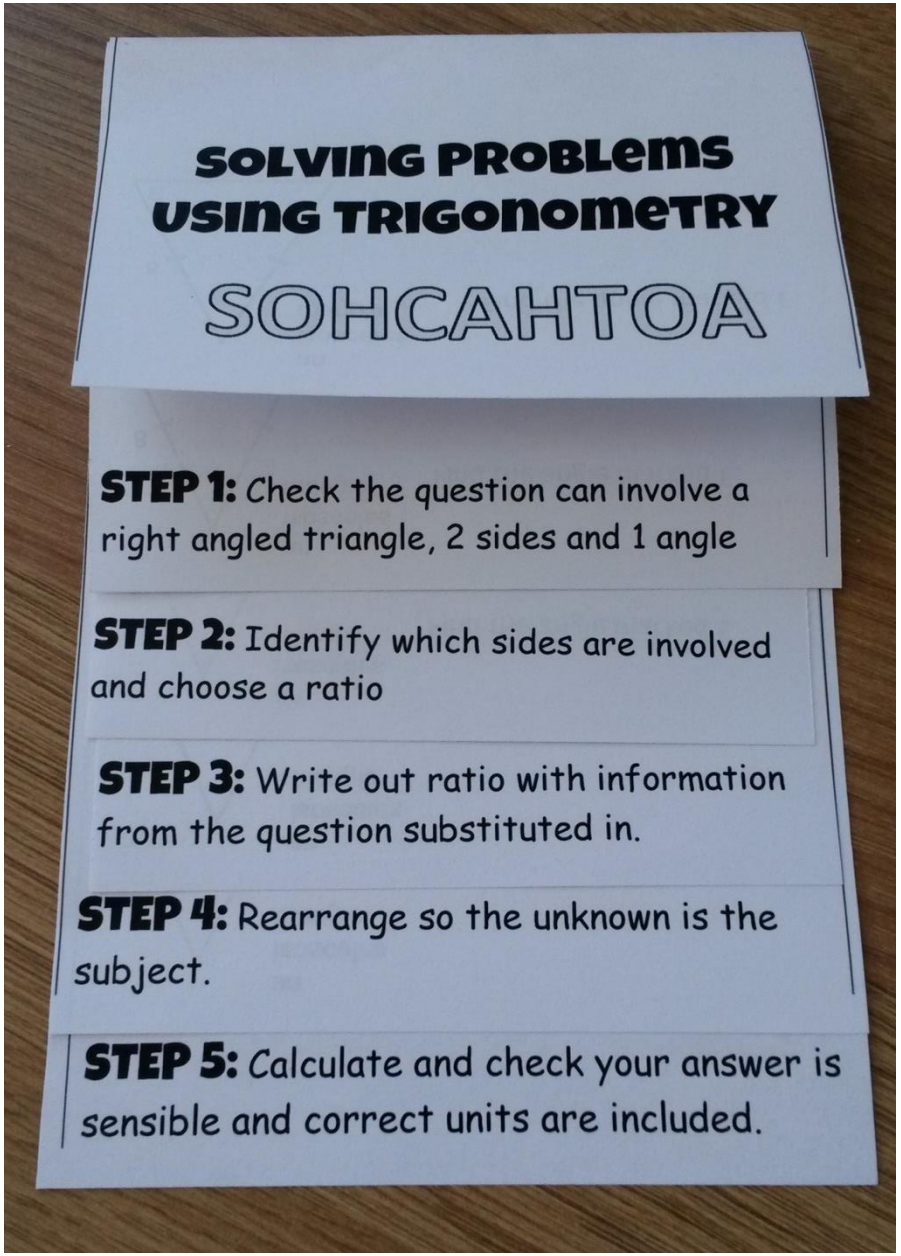
sin 53.13° = 0.8
e = ?



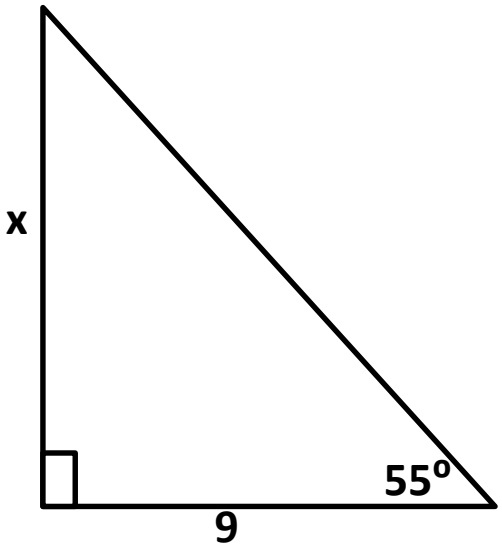
cos 60° = 0.5
f = ?

I do love a foldable!!

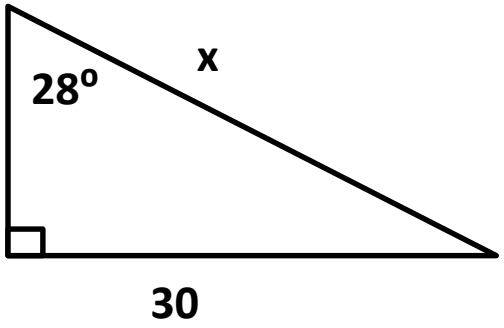
Taking students from understanding to a method they can use…without removing the understanding??



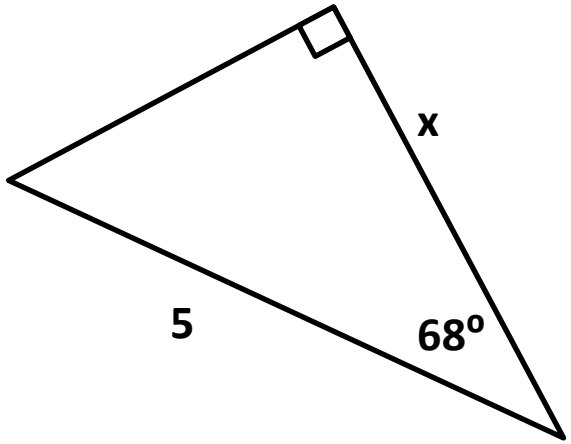
What is the missing side x? (leave to 2.d.p.)



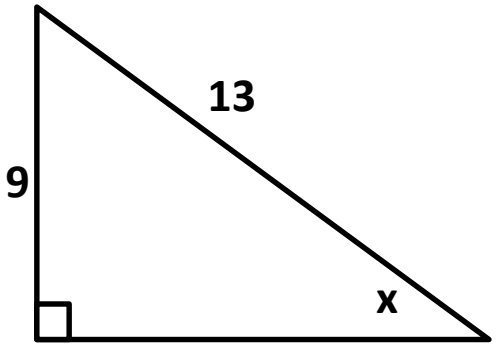
What is the missing side x? (leave to 2.d.p.)



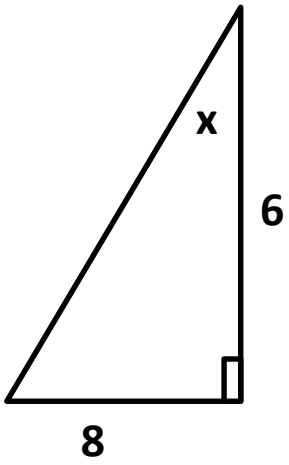
What is the missing side x? (leave to 2.d.p.)



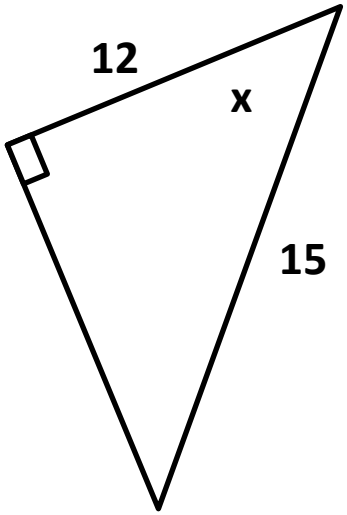
What is the missing angle x? (leave to 2.d.p.)



What is the missing angle x? (leave to 2.d.p.)



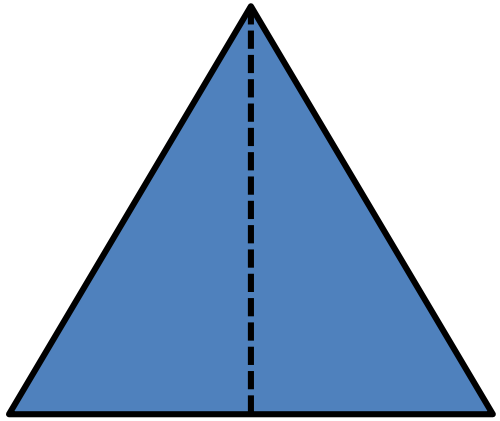
What is the missing angle x? (leave to 2.d.p.)



A ladder 15m high is placed 4m away from the foot of a wall. What angle is the ladder placed at to the ground?	A ship sails on a bearing of 035° for 28km. How far North has the ship travelled?	A ship sails on a bearing of 035° for 28km. How far East has the ship travelled?
After a days walking, a man is 5km North away from his start point, and 6km East. What bearing did the man walk on?	A girl is flying a kite on a string 32m long. The string, which is being held at 1m above the ground, makes an angle of 39° with the horizontal. How high is the kite?	James stands about 50m away from a building. The angle of elevation from James to the top of the building is about 15°. How tall is the building?

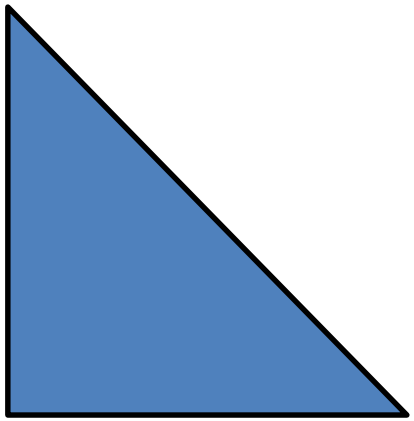
Exact trig ratios

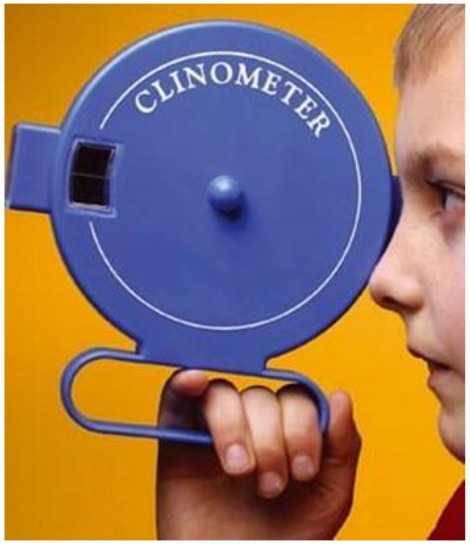
An equilateral triangle with side length 2



Angle (θ)	sin(θ)	cos(θ)	tan(θ)
0°			
30°			
45°			
60°			
90°			

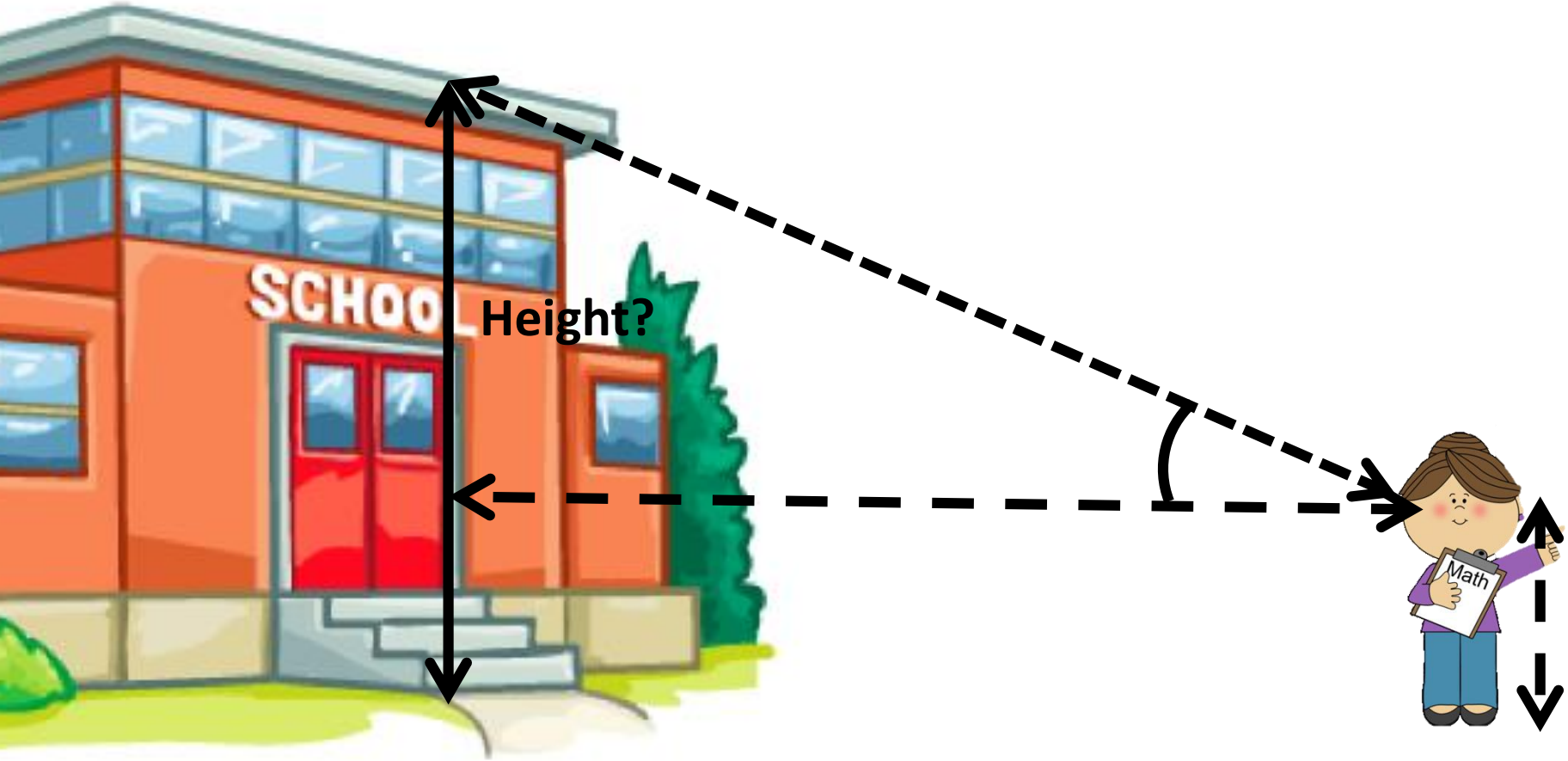
A right angled isosceles triangle with shortest sides 1

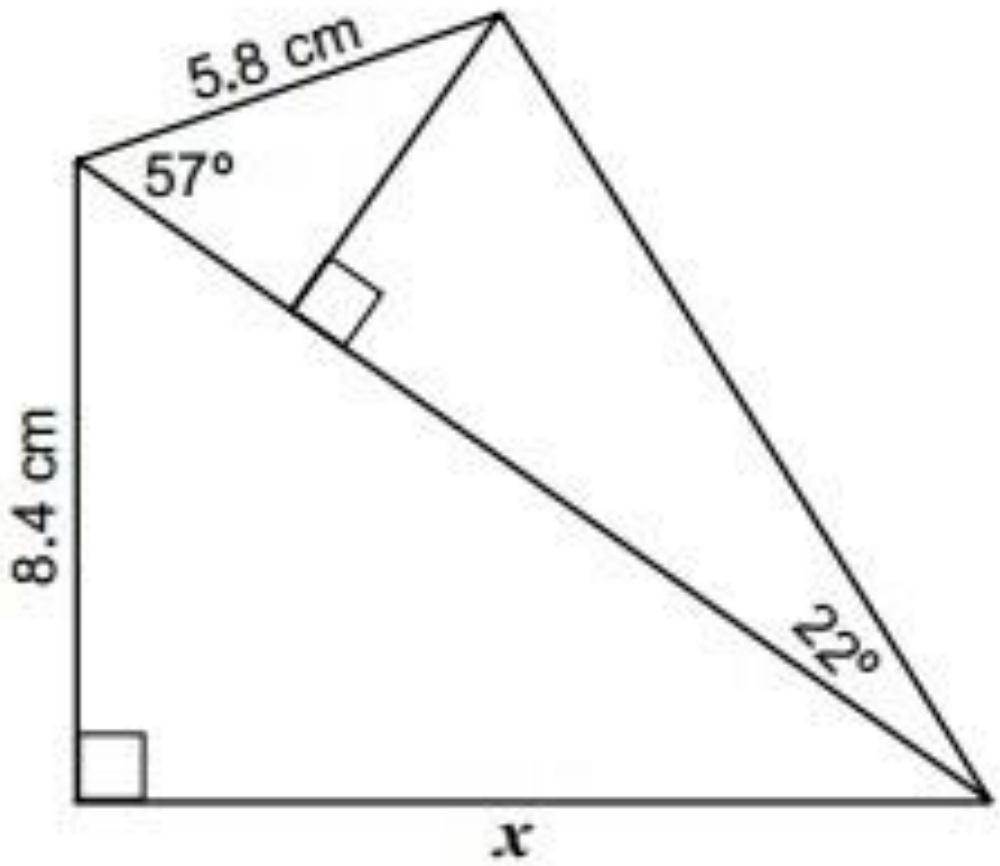




clinometers

Using a Clinometer

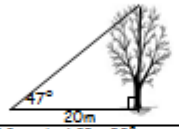


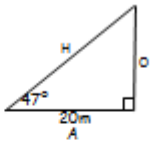

5.8 cm
57°
8.4 cm
22°
x

Find the 4 mistakes!

A

From a point on the ground 20m from the base of a tree, the angle of elevation to the top of the tree is 47°. Calculate the height of the tree.





Tan 47 = O/A

Tan 47 = O/20

20 x tan 47 = O

O = -2.5 m (1d.p.)

B

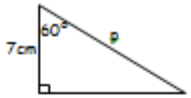
In ΔABC, angle ACB = 90°, angle BAC = 47.6° and AB = 4.6cm. Find BC.

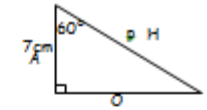
C

An isosceles triangle has two equal sides of length 8cm and two equal angles of 60°. calculate the height of this triangle?

D

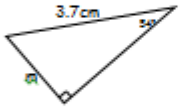
Find the length of side p.





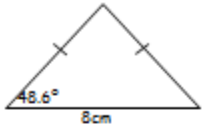
Cos 60 = A/H

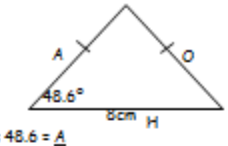
E



F

Calculate the perimeter of this isosceles triangle.





Cos 48.6 = A/H

Cos 48.6 = A/8

8 x cos 48.6 = A

A = 5.3 cm (1.d.p.)

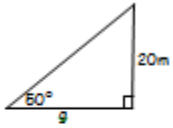
Perimeter = 5.3 + 5.3 + 8 = 18.6 cm

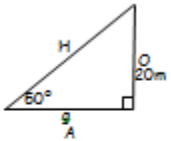
G

From a point on the ground 10m from a block of flats the angle of elevation to the top of the block is 76°.
Calculate the height of the block of flats.

H

Calculate the size of g.





Tan 60 = O/A

Tan 60 = 20/g

g = 20 x tan 60

g = 23.8 m (1d.p.)

Spaghetti Trigonometric Graphs