

Problem Solving



National
Teaching
School



Wymondham
College



The Inclusive Teaching School



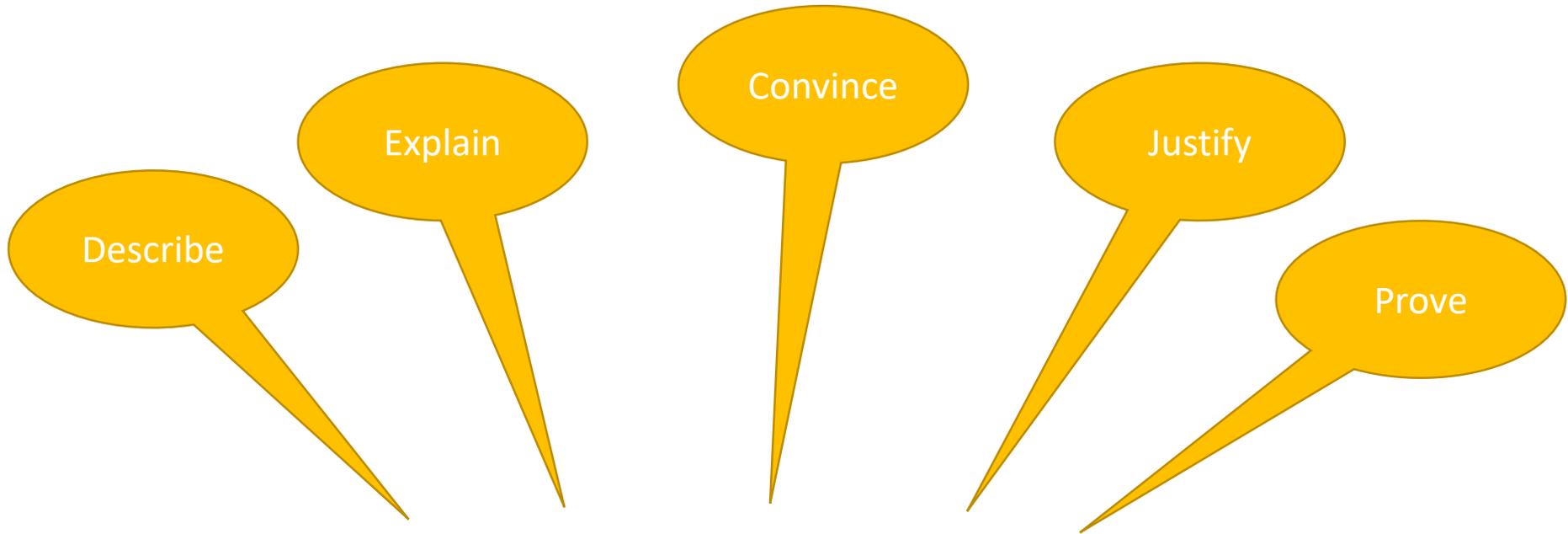
Farlingaye High School

Aims of this session – teacher will:

- Reasoning tasks discussion from last session
- Reading discussion
- Problem solving
- What is problem solving?
- Problem solving skills.
- Problem solving tasks.



Reasoning



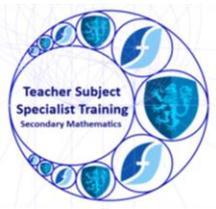
Novice to Expert

Homework from previous session

Use one of the NRIC tasks with your pupils.

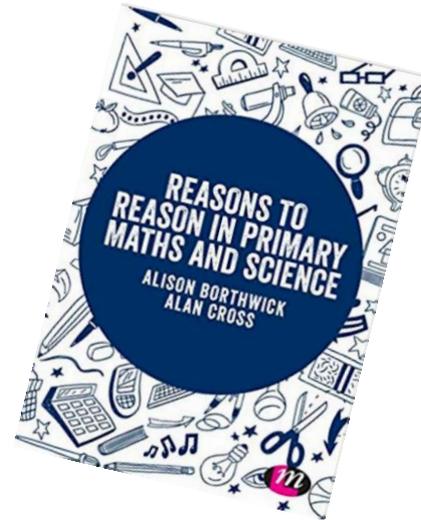
Did you use the grid? Which of the types of reasoning did you observe?

Convincing	Working abstractly	Decomposing	Interpolating	Evaluating
Being logical	Hypothesising	Extrapolating	Proving	Predicting
Deconstructing	Generalising	Speculating	Explaining	Agreeing/ disagreeing
Questioning	Noticing	Describing	Justifying	Making connections
Recognising links	Being systematic	Making judgements	Pattern seeking	Exemplifying
Comparing	Doing / undoing	Organising	Refuting	Tinkering
Correcting	Altering	Specialising	Verifying	Deleting
Offering counter examples	Visualising	Working backwards	Wondering	Puzzling
Classifying	Observing	Sorting	Conjecturing	playing



Reasoning

- Reason to Reason Book Chapter 3 – Reasoning in the Early Years.
- Thoughts?



What is problem solving ?

How does it fit with the mathematics national curriculum?

How does it fit with the mathematics scheme of work in your school?

What is your definition?



What does the National Curriculum say:

Children:

Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down into a series of simpler steps and persevering in seeking solutions.



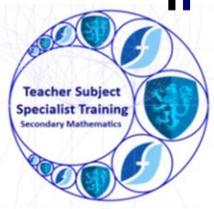
The importance of problem solving

• YFS - both strands have problem solving as an integral part

• PS is not just about solving puzzles. It is about *how* you solve them; how do you start, continue, finish?

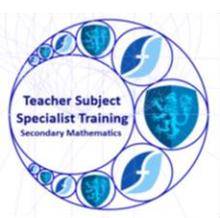
• Problem solving is not just for HAPs or those who finish first!

• Problem solving does not just happen on a Friday!



Problem Solving

- ‘To be successful in both mathematics and science, learners need to construct their understanding through solving problems and investigating.’ Cross and Borthwick
- ‘ To use logical reasoning, suggest solutions and try out different approaches to problems’ Haylock 2010
- “the mathematician’s main reason for existence is to solve problems [...] therefore, what mathematics really consists of is problems and solutions” (Halmos, 1980, p. 519, original emphasis).



A bit of Maths...

Find the number, given the following clues:

- It's less than 100
- It's one more than a multiple of 3
- Exactly one of its digits is a prime
- If you reverse its digits, you get a prime
- It's not a multiple of 5
- It's not prime
- It has exactly 4 factors
- It's not square
- The sum of its digits is prime
- If you multiply it by 5, the answer is greater than 100



How do you develop problem solving skills in pupils?

How do you teach pupils the thinking skills necessary to solve mathematical problems in contexts that are unfamiliar to them?



Different Types of Problem Solving

The Primary National Strategy (May 2004) suggested that there are five different types of problem:

- Finding All Possibilities (for example [Half Time](#))
 - Logic (for example [Teddy Town](#))
 - Visual (for example [Baravelle](#))
 - Rules and Patterns (for example [Ip Dip](#))
 - Word Problems
-
- Think and discuss
 - same skills, different types of problems
 - Same types of problems, different skills



Skills /strategies of problem solving

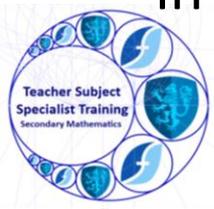
- Predicting, conjecturing and hypothesising
- Visualise - Using diagrams and pictorial information
- Work backwards
- Pattern spotting
- Grouping and classifying
- Working systematically
- Looking for all possibilities
- Reason logically
- Interpreting word problems
- Trial and improvement
- To inquire
- Using manipulatives
- Generalising



- **What could problem solving look like in a primary maths classroom?**

In his chapter on thinking mathematically (1992), Alan Schoenfeld suggests that whilst the idea of *problems* has been a part of the maths curriculum for ever, *problem solving* has not. And furthermore there are different definitions of what a problem is, and hence what problem solving means.

At one extreme we have sets of 'problems' which are all about practising a technique. In the classroom this typically involves the teacher introducing a task and illustrating the technique, and then the children do lots more 'problems' on the same theme so that they master the technique which becomes part of their mathematical toolkit. Problem solving is interpreted as working through a series of related and predictable questions in order to acquire a particular skill.



- However an alternative interpretation is that of **Polya (1945)**. Problem solving in Polya's view is about engaging with real problems; guessing, discovering, and making sense of mathematics. (Real problems don't have to be 'real world' applications, they can be within mathematics itself. The main criterion is that they should be non-routine and new to the student.) Compared to the interpretation as a set of questions on a theme, Polya's is a much more challenging interpretation of problem solving for a teacher to come to terms with, but has the potential to be much more effective in developing young mathematicians who have an **'understanding of the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject'**.

For Polya (1945), problem solving is:

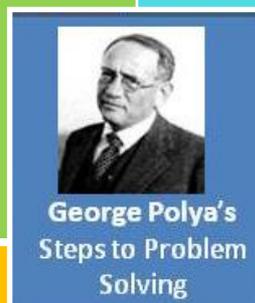
- seeking solutions not just memorising procedures
- exploring patterns not just memorising formulas
- formulating conjectures, not just doing exercises.



Step One

Understand the Problem

Can you state the problem in words?
What are you trying to find or do?
What are the unknowns?
What information do you obtain from the problem?
What information, if any, is missing or not needed?



Step Two

Make a Plan

Look for a pattern.
Remember related problems.
Break the problem down into different parts.
Make a table.
Make a diagram.
Write an equation.
Use a guess and check.
Work backward.
Identify a subgoal.



Step Three

Do the Plan

Implement the strategy in Step 2 and perform the necessary maths calculations.
Check each step of the plan as you do it.
Keep an accurate record of your work.
Organise your work into easy to understand visuals.
Double check your maths work.



Step Four

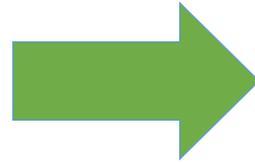
Look Back

Check the results in the original problem.
Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable?
Determine whether there is another method of finding the solution.
If possible, determine other related or more general problems for which the techniques will work.

A Problem Solving Process

Getting started:

Try a simpler case
Draw a diagram
Represent with a model
Act it out



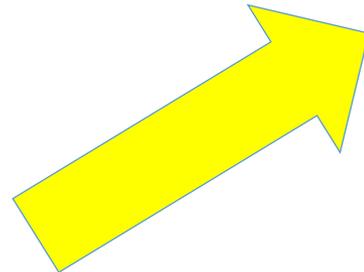
Working on the problem:

Visualise
Work backwards
Reason logically
Make a conjecture
Work systematically
Look for a pattern
Trial and improvement



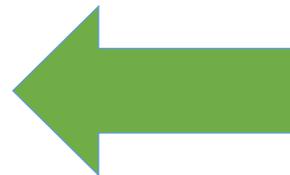
Going further:

Generalise
Justify
Verify
Prove it

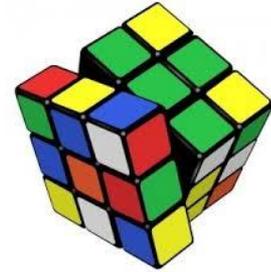


Concluding:

Communicating findings
Evaluate and review

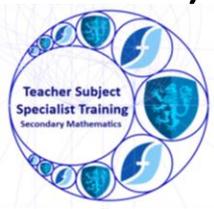


Strategies for Problem Solving



There are numerous methods and strategies to solve maths problems. These are some of the most common strategies :

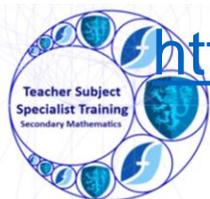
- 1) Draw a picture
- 2) Look for a pattern
- 3) Guess and check (Trial and Error)
- 4) Make a systematic list
- 5) Logical Reasoning
- 6) Work backwards



Mindset and Problem solving

'It is important not to provide too much help to student and take away from the cognitive demands of tasks...students expect to be helped, and teachers know it is their role to help them, so the teachers break down the problem and make it easier. In doing so they empty the problem of its cognitive demand. When my students ask for my help, I am very careful not to do the mathematical thinking for them.'

Mathematical Mindsets Jo Boaler.



<https://www.youtube.com/watch?v=67AxLqrdDLQ>



Roll the dice. Add up the numbers on the 2 red dice and subtract the number on the green dice.

Find all the possible answers you can make.

How will you know if you are successful?

How will you record what you have found out?

9	16
25	43

NUMBER 1

Which is the odd one out and why?



Making the most out of a problem

Hannah has lots of 2p and 5p coins, How many ways can she make 30p?



Making the most out of a problem

- Removing intermediate steps
- Reversing the problem
- Making the problem more open
- Asking for all possible solutions
- Asking why, so that pupils explain
- Asking directly about a mathematical relationship

“Children don’t become problem solvers by doing problems”

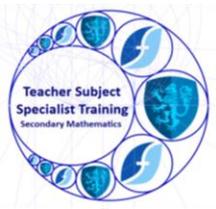
(Jane Jones)



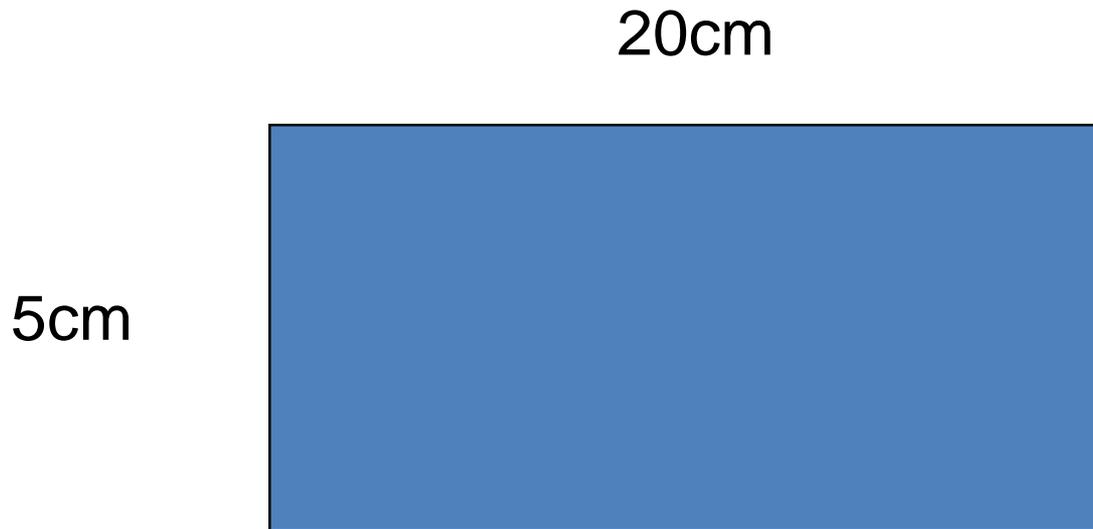
Making the most out of a problem

- Changing Questions

Question from class/learner	Better version?	Better still?
What is $3+4$?	What pairs of numbers make 7?	What pairs of numbers when operated result with the answer 7?
Which ball rolls best?	Which ball rolls furthest?	How does the size of the ball affect the distance it rolls?



Making the most out of a problem



Adapt this question to encourage pupils to think harder about how to solve it, and better develop problem solving and conceptual understanding of area of rectangles.

Problem Solving Worksheet

At a restaurant the charge for a 3 course meal was £40 for an adult and £25 for a child. Mrs Davies paid a total of £755 for a party of 20 people. How many children were at the dinner party?

There were some passengers on the bus as it left station A. At Station B 15 passengers got off and 20 got on. At Station C, 9 passengers got on and none got off. Then there was 45 passengers on board. How many passengers were on the bus at Station A?

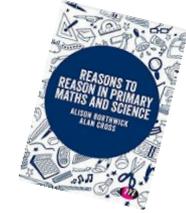
Find the value of X and Y

A	A	A	27
B	A	B	25
A	B	A	X
B	B	B	Y

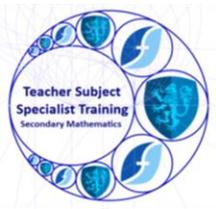
Solve each problem using two different strategies



Homework

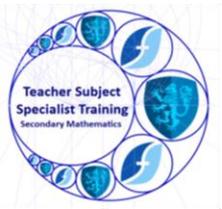


- Read Chapter 7: Using problem solving and investigations to promote reasoning
- Read the article: Developing excellence in Problem Solving with young learners by Jennie Pennant and the NRICH team.
<https://nrich.maths.org/10865>
- Read the article: Using NRICH tasks to develop key problem solving skills by Liz Woodham. <https://nrich.maths.org/11082>
- Try out different types of problem solving activities – routine and non-routine - Record the type of skills/strategies your pupils are using to solve these problems. Are they using the same strategy for all problems?



Contact Details

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Homework

Activities which develop Problem Solving

Try at least two of the activities on the list or from today's session, with one or more of your classes.

Record the type of skills/strategies your pupils are using to solve these problems. Are they using the same strategy for all problems?

Read the next chapter from Reasons to Reason – Questions to Promote Reasoning.

