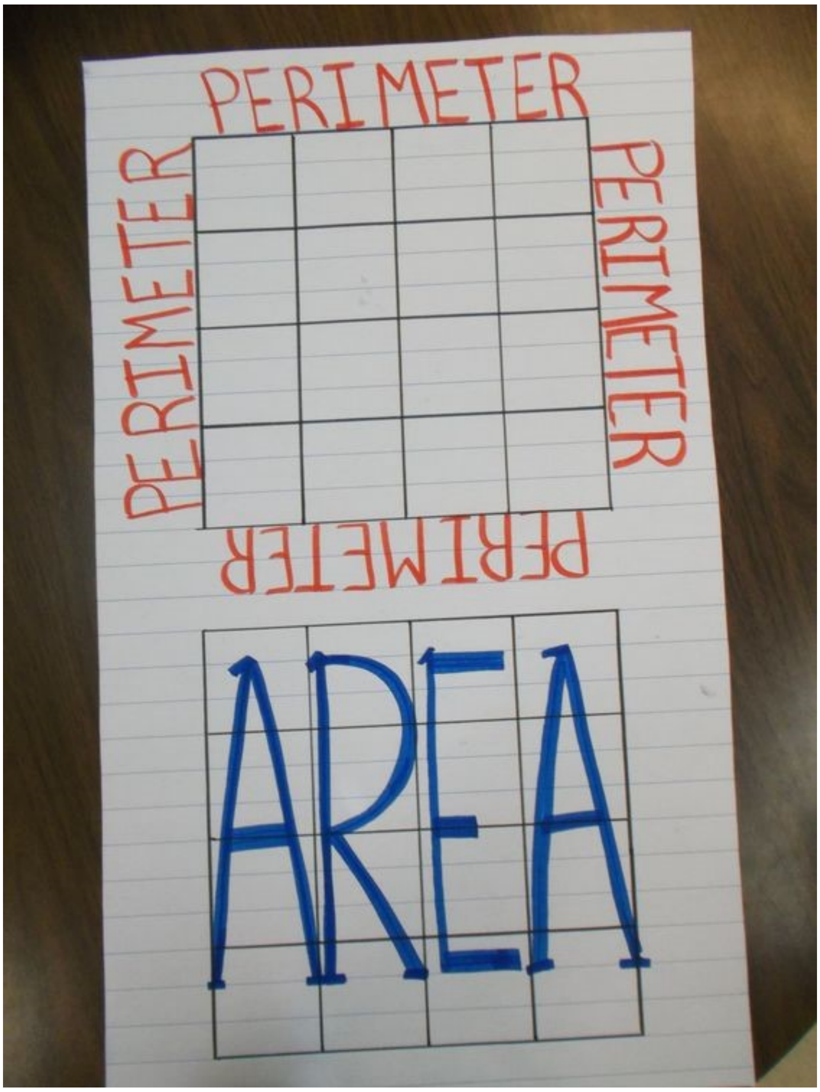
Measures



Understanding perimeter, area and volume and their interconnections

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Perimeter

Websites for resources

- https://nrich.maths.org/
- www.khanacademy.org
- Circle song - https://www.youtube.com/watch?v=-z4SUypJZxo

Why we think this is a useful activity

A good starting point is counting squares and counting the perimeter length. This helps understanding of what area and perimeter are.

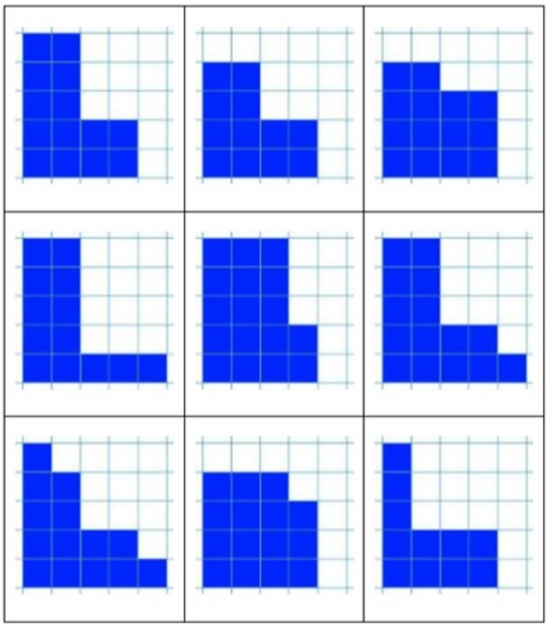
Misconceptions

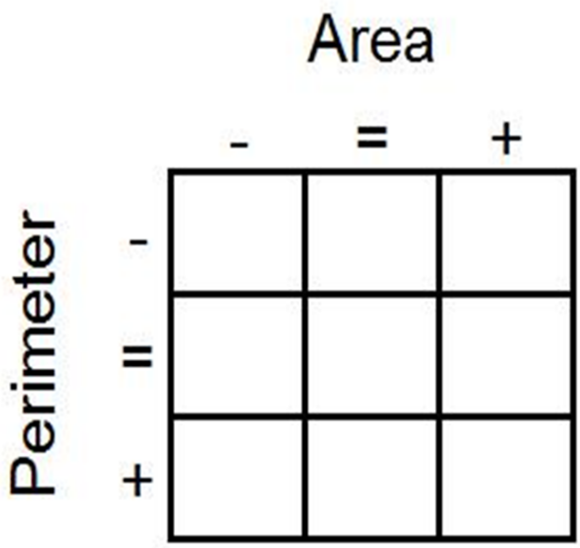
⇒ Students believe area is bigger than perimeter since it involves multiplying.

⇒ Students confuse area and perimeter.

⇒ Students count the perimeter incorrectly, using points or squares around the outside instead of lengths.

Nrich - Changing areas, changing perimeters

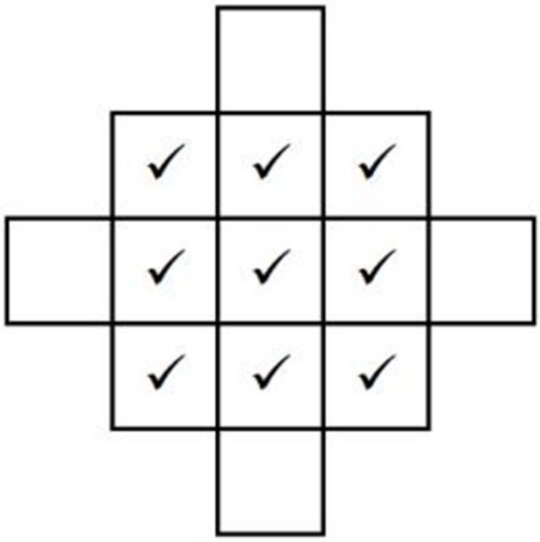




As you go from left to right, the area of the shapes must increase.
As you go from top to bottom, the perimeter of the shapes must increase.
All the shapes in the middle column must have the same area.
All the shapes on the middle row must have the same perimeter.

What reasoning can you use to help you to decide where each card must go?

2 by 8 rectangle	4 by 4 square	1 by 15 rectangle
5 by 5 square	3 by 8 rectangle	2 by 7 rectangle
1 by 16 rectangle	3 by 6 rectangle	1 by 9 rectangle

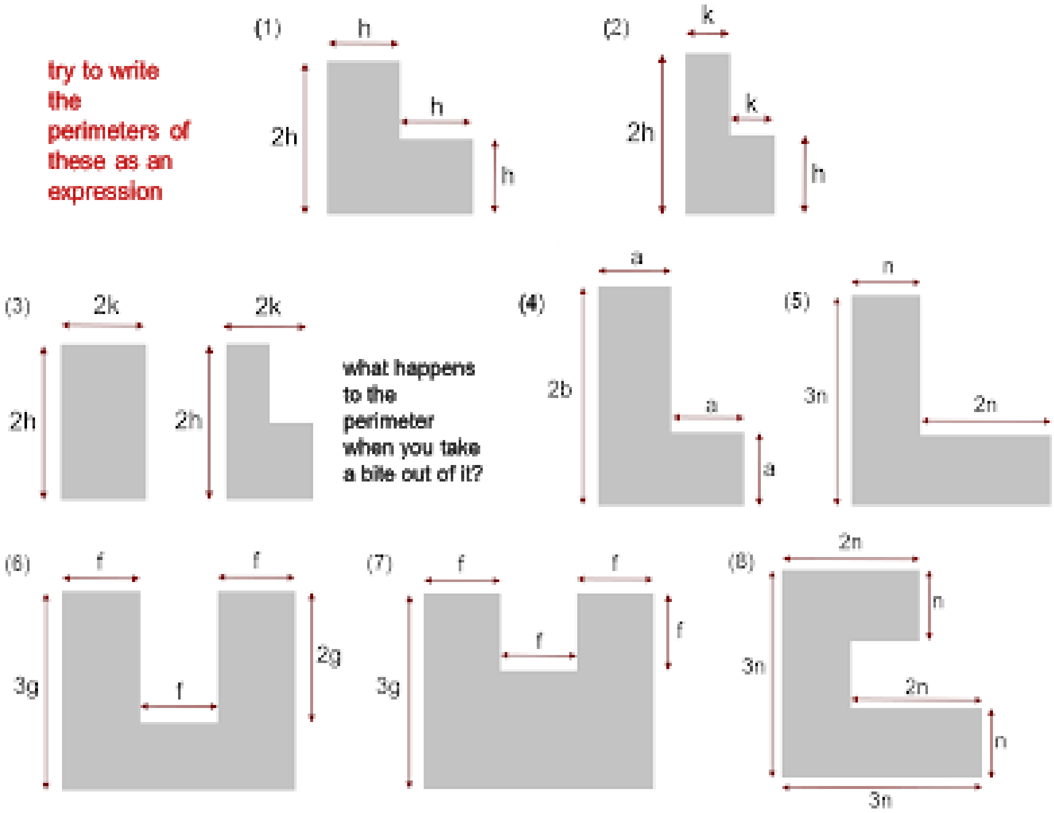


Can you arrange them in the grid in the same way?

The ticks represent the nine cards you've already placed. Can you create cards with dimensions for rectangles that could go in the four blank spaces that satisfy the same criteria?

Not all the spaces are possible to fill. Can you explain why?

L shaped perimeters - (Don Steward)



A nice activity involving algebra. Students need to calculate the missing lengths using the lengths provided. Then students can calculate the perimeter of the rectilinear shapes.

This activity involves algebra but could be made easier using numbers instead.

Area

Websites, resources & activities

- https://nrich.maths.org/
- Standards unit SS2 and SS4
- https://mrbartonmaths.com/teachers/rich-tasks/standards-units.html
- www.resourceaholic.com
- https://donsteward.blogspot.com/
- www.greatmathsteachingideas.com

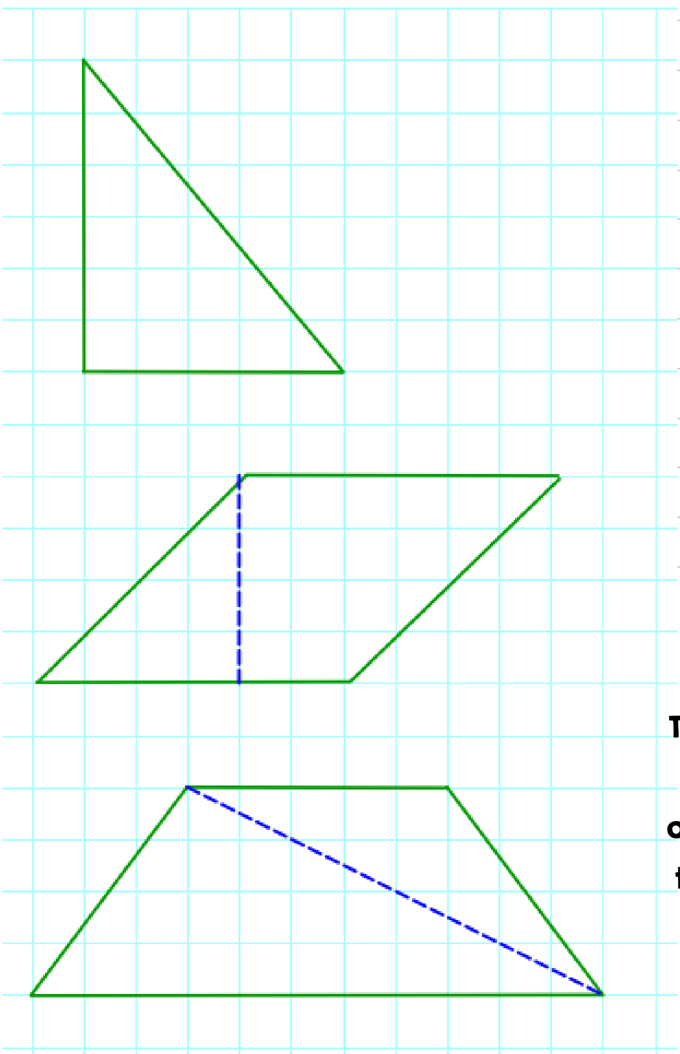
Misconceptions

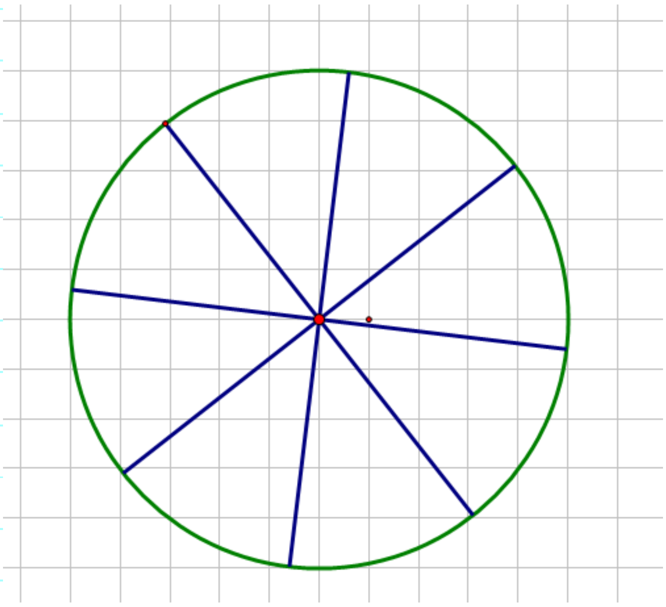
⇒ Students learn the formulae without understanding its meaning, and as a consequence use them incorrectly.

⇒ Students forget to divide by 2 for area of a triangle.

⇒ Students use the wrong measurement for height in triangles, parallelograms and trapeziums.

Visual representations

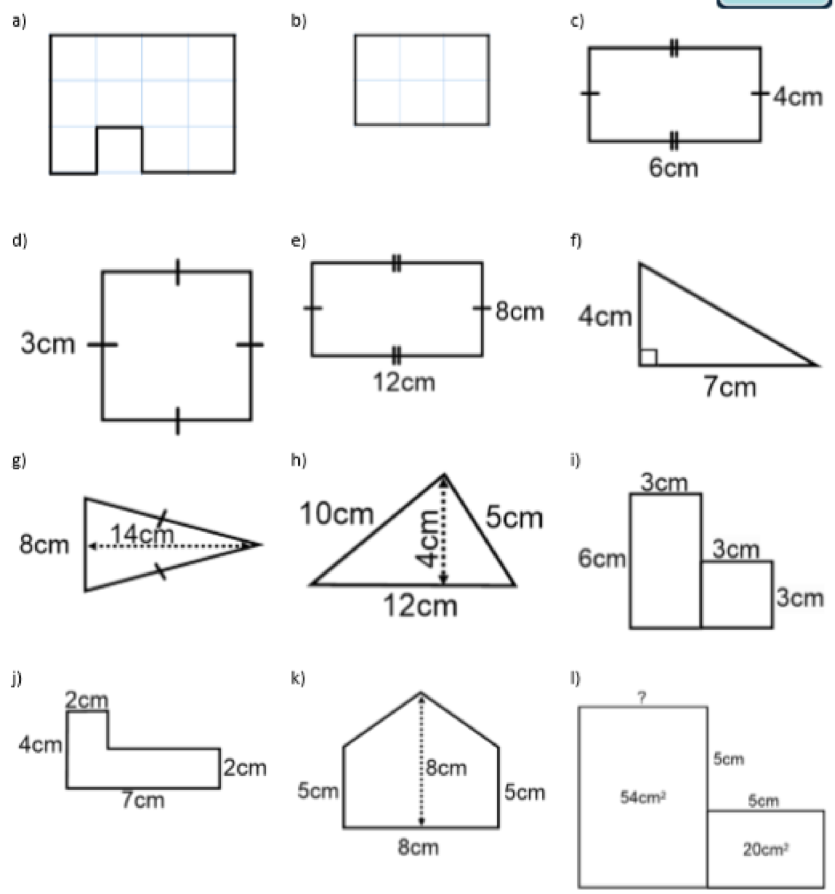




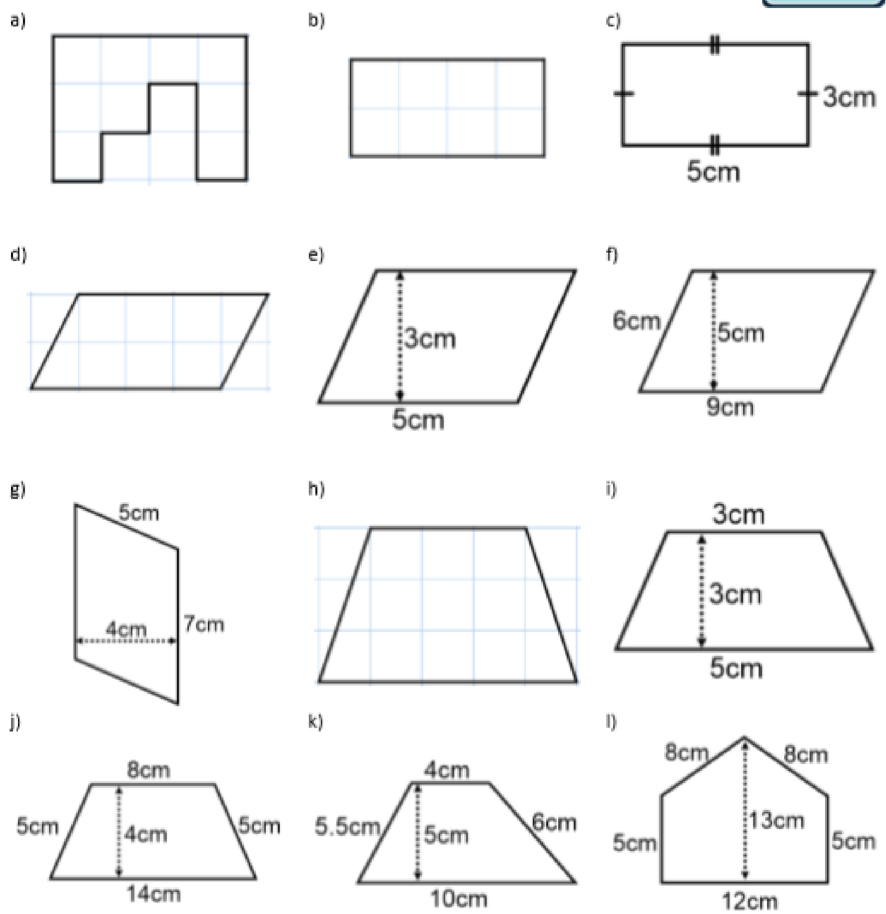
Why do we think this is worth doing?

This activity uses our knowledge of area of rectangles to explain the area for other 2D shapes. It is a nice practical activity to engage learners and give them a strong foundation before attempting to solve problems on area.

Area of 2D Shapes



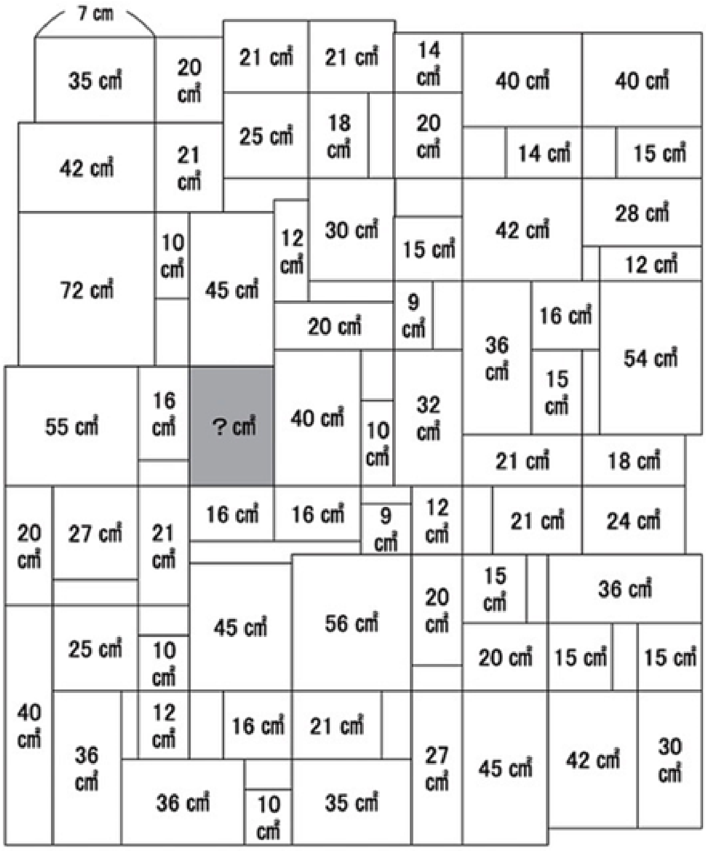
Area of 2D Shapes II



Increasing difficulty questions

Each question gets slightly more difficult and so eases the students into more challenging problems. These resources were found on resourceaholic and there are resources like this for most mathematical topics.

Don steward - area mazes

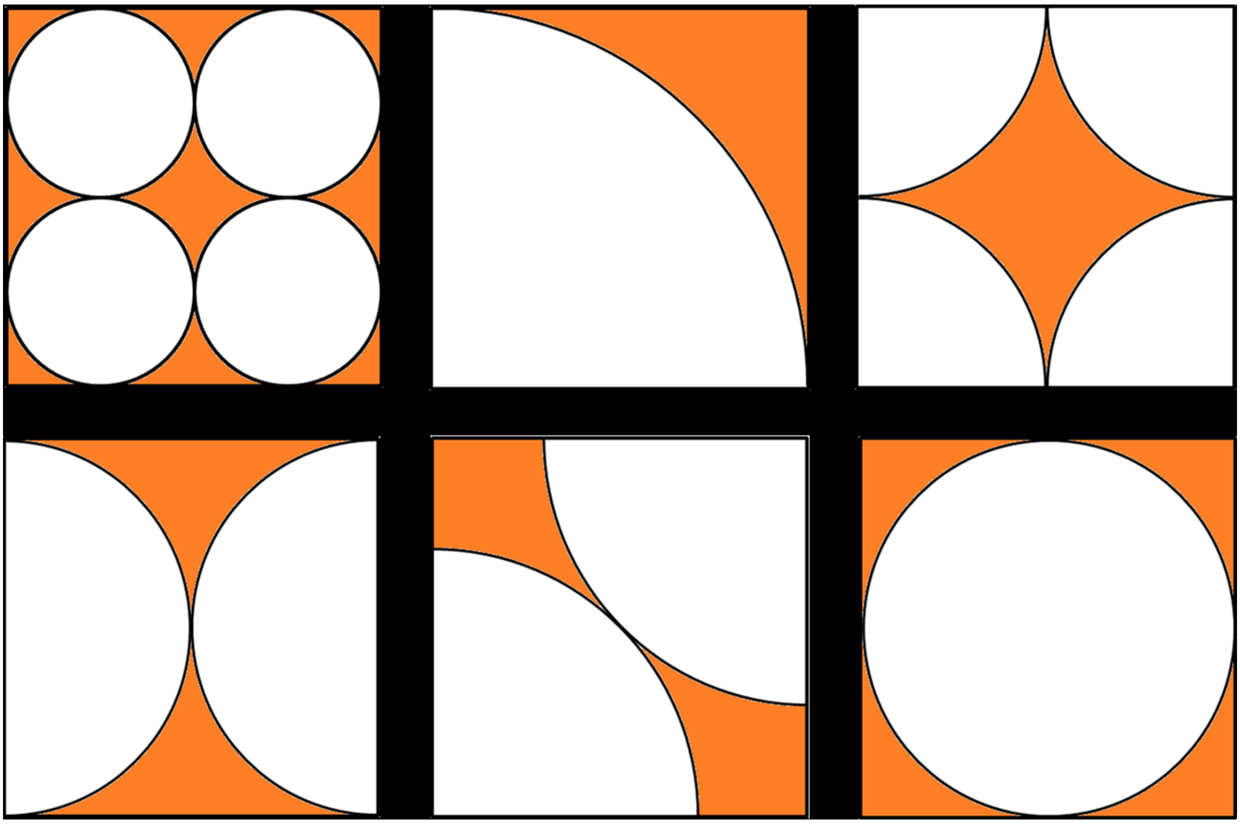


This task involves working backwards from the area to calculate the lengths.

Starting point 35 divided by 7.

This puzzle ensures students check their work to avoid making a mistake that will then follow through the entire task.

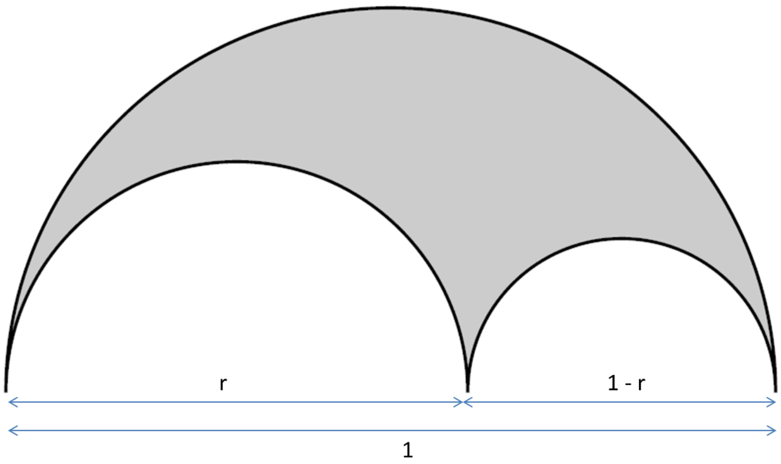
Curvy areas



Each square should be the same size. (i.e. 5cm x 5cm)

Then students calculate the area of the shaded sections in each diagram using their knowledge of area of a circle. All circles are half circles, quarter circles or full circles. They should notice something about their answers. (Extension—leave answers in terms of pi)

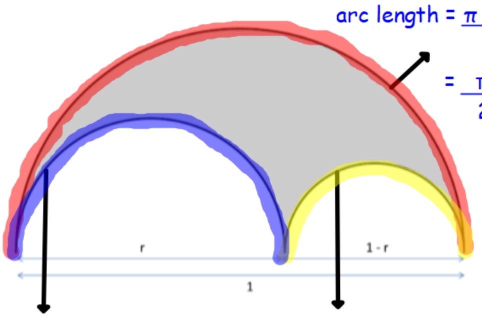
The Arbelos



Task: To find the maximum area of the Arbelos.

Give each student a piece of paper and construct the Arbelos with particular dimensions. (Give each student a different value for r.) Plot the results on a graph and discover which value of r gives the maximum area.

After investigating with different sizes give the students the opportunity to prove the results using algebra.



arc length = $\frac{\pi \times \text{diameter}}{2}$

$= \frac{\pi}{2}$

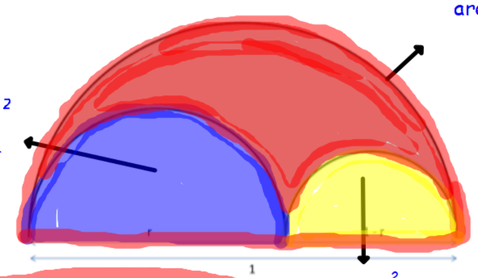
arc length = $\frac{\pi \times \text{diameter}}{2}$

$= \frac{\pi r}{2}$

arc length = $\frac{\pi \times \text{diameter}}{2}$

$= \frac{\pi(1-r)}{2}$

Total perimeter = $\frac{\pi + \pi r + \pi(1-r)}{2} = \pi$



area = $\frac{\pi r^2}{2}$

$= \frac{\pi \times \left(\frac{r}{2}\right)^2}{2}$

$= \frac{\pi r^2}{8}$

area = $\frac{\pi r^2}{2}$

$= \frac{\pi \times \left(\frac{1}{2}\right)^2}{2}$

$= \frac{\pi}{8}$

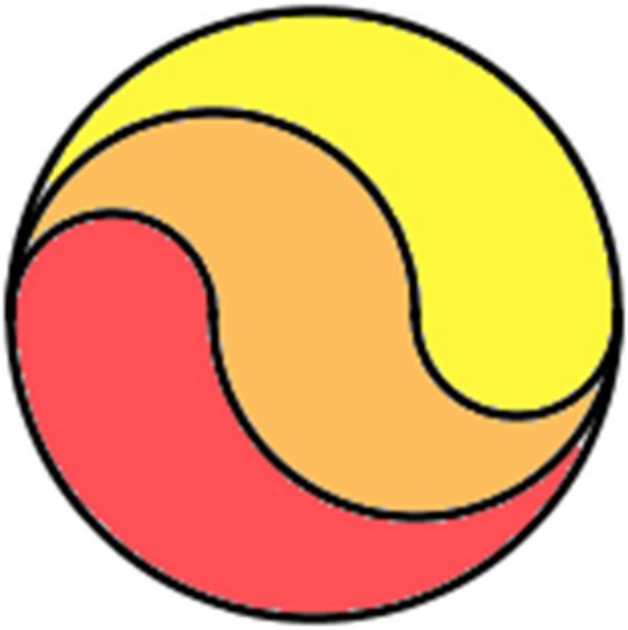
area = $\frac{\pi r^2}{2}$

$= \frac{\pi \times \left(\frac{1-r}{2}\right)^2}{2}$

$= \frac{\pi - 2\pi r + \pi r^2}{8}$

$\boxed{\begin{aligned} &\text{Total area} = \text{red} - \text{blue} - \text{yellow} \\ &= \cancel{\tfrac{\pi}{8}} - \tfrac{\pi r^2}{8} - \cancel{\tfrac{\pi}{8}} + \tfrac{2\pi r}{8} - \tfrac{\pi r^2}{8} \\ &= \tfrac{\pi r}{4} - \tfrac{\pi r^2}{4} \\ &= \tfrac{\pi r(1-r)}{4} \end{aligned}}$

Curvy areas



Task: To find the area of each coloured section.

This is a similar problem to the arbelos, using semi circles to solve it.

Group students into teams of three and give them a section to each work out.

What do you notice?

Volume

Websites, resources & activities

- https://nrich.maths.org/
- www.resourceaholic.com
- http://www.greatmathsteachingideas.com/tag/investigation/

Why we think this is a useful activity

Students predict which tray they think will hold the most. Most students quickly choose the tallest but then realise that this is not the case. A nice practical activity to introduce volume.

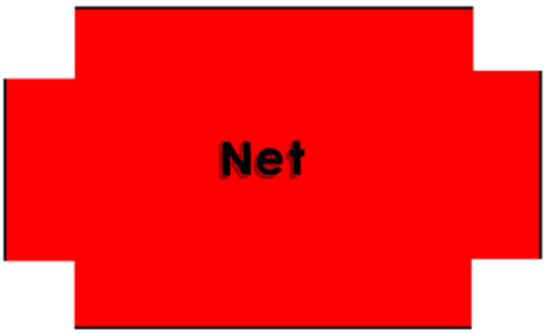
Misconceptions

- ⇒ Wrong use of units used.
- ⇒ Students believing if an object is taller then it has a larger volume.
- ⇒ When calculating the surface area, students only calculate the area of the visible faces.

Max box

Make a tray out of a piece of A4 paper using the measurements given to you.

Calculate the volume of your tray



Max box

NRICH task—Cuboid challenge is similar to this problem.

	Dimensions of box			
Size of corner square?	Length	Width	Depth	Volume

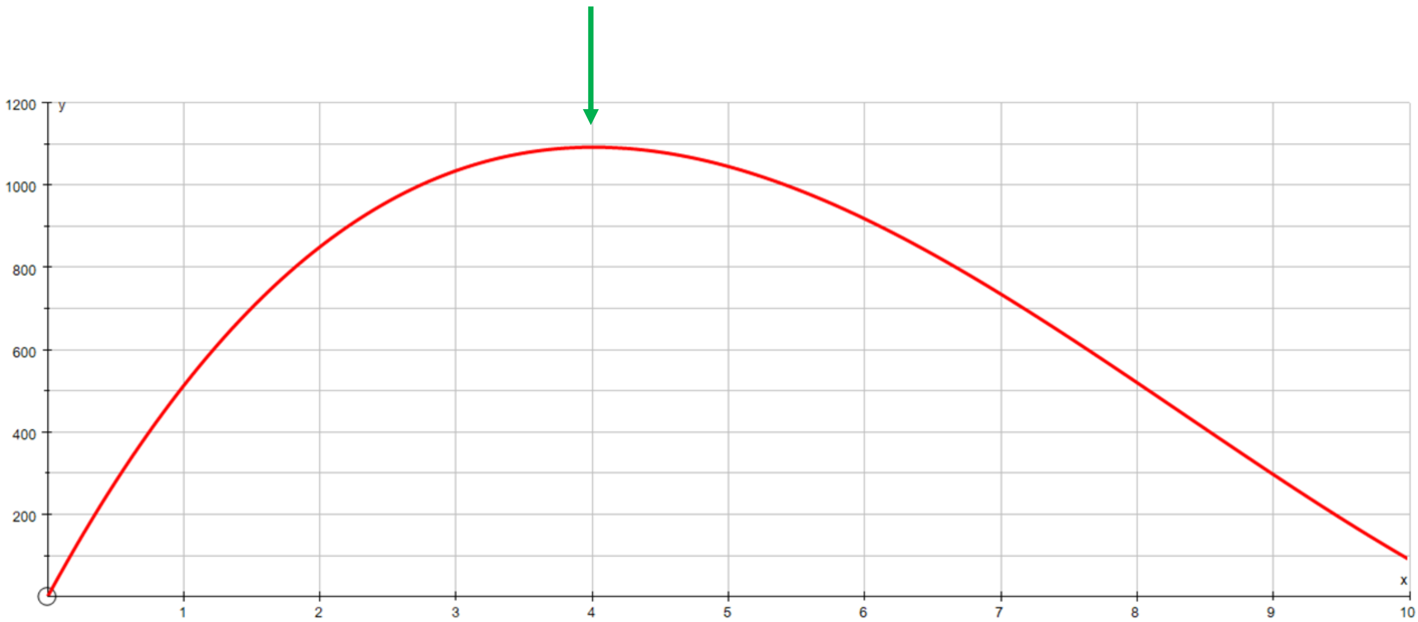
Complete the table with the dimensions of your tray and collect results from others. Plot a graph to show these results.

What size corner square would give the largest volume?

For a4 piece of paper

$y = x(21-x)(29-x)$

Maximum point



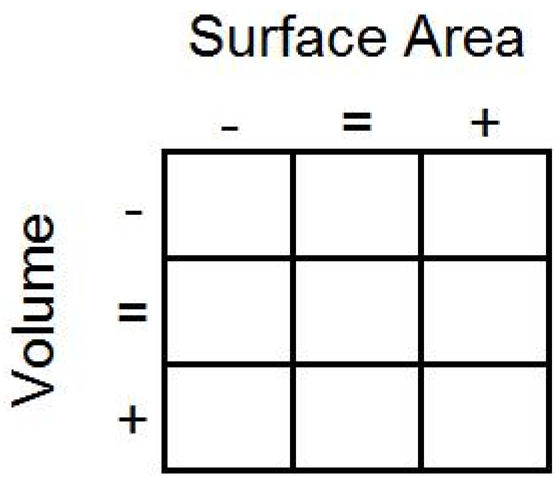
How to extend further

Now try starting with different sized square sheets of paper.

Can you find a relationship between the size of paper and the size of the square cut-out that produces the maximum volume?

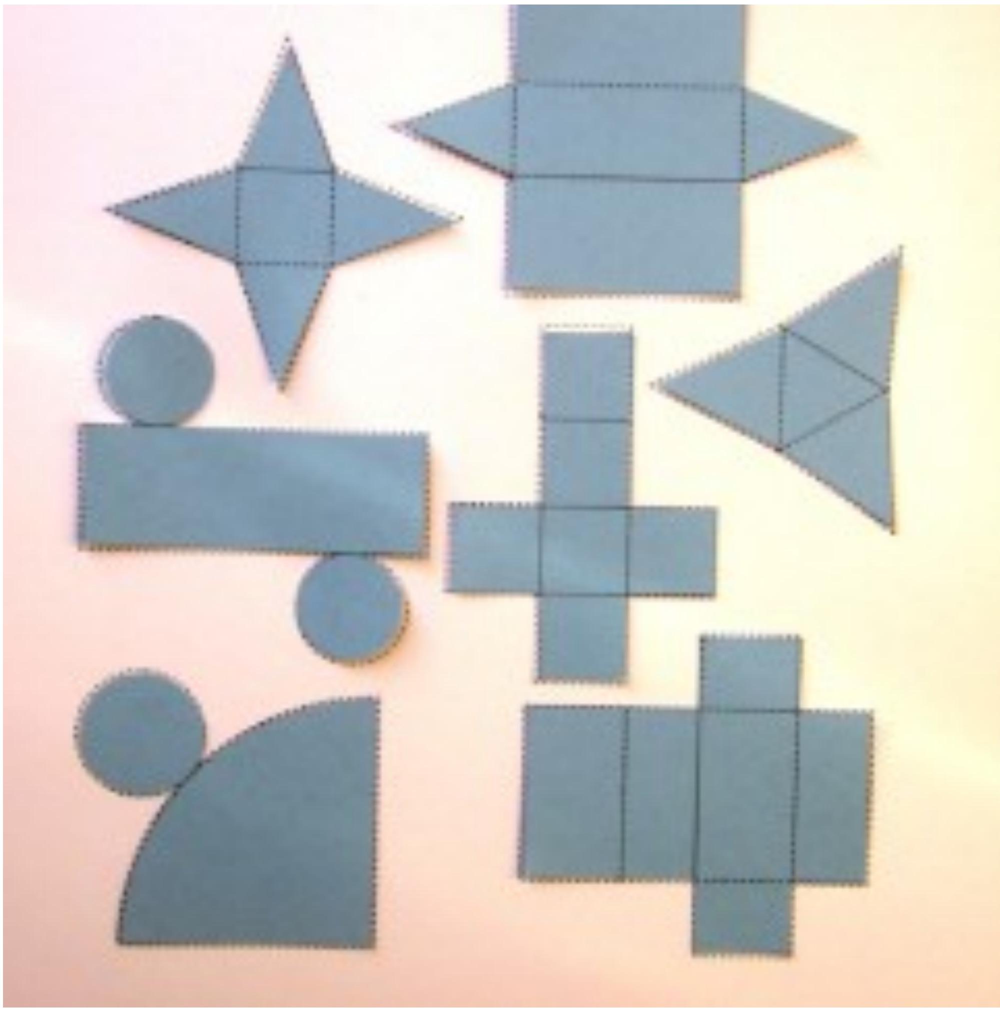
Nrich - changing areas, changing volumes

1 by 2 by 28 cuboid	4 by 4 by 4 cube	2 by 4 by 7 cuboid
1 by 2 by 26 cuboid	2 by 4 by 6 cuboid	4 by 5 by 6 cuboid
4 by 5 by 7 cuboid	1 by 2 by 24 cuboid	1 by 4 by 14 cuboid



As you go from left to right, the surface area of the shapes must increase.
As you go from top to bottom, the volume of the shapes must increase.
All the cuboids in the middle column must have the same surface area.
All the cuboids on the middle row must have the same volume.

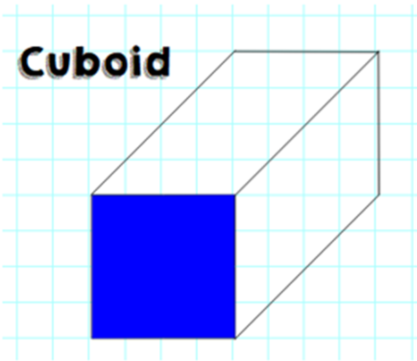
Surface area nets

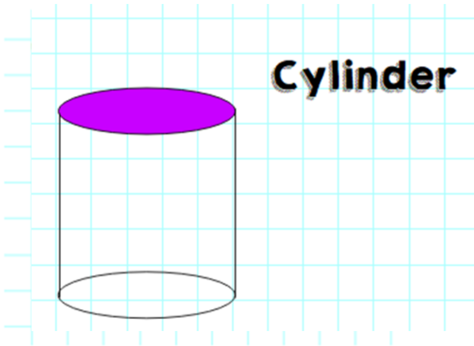


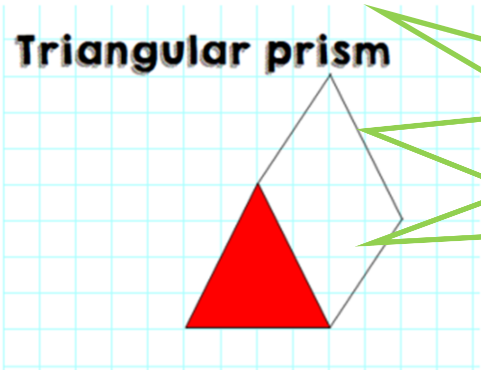
When introducing how to calculate the surface area of objects, use nets .

Surface area challenge - can you create a cuboid with a surface area of $60cm^2$? How many different possible cuboids are there?

Prisms







Volume of prisms

Area of cross section x depth

Cones, spheres and pyramids

Students need to be able to substitute into the provided formulas.

Volume of cone = $1/3 \times \pi \times r^2 \times h$

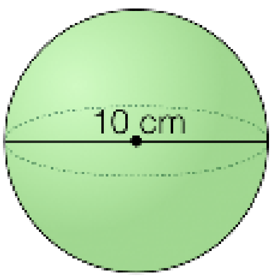
Volume of sphere = $4/3 \times \pi \times r^3$

Volume of pyramid = $1/3 \times$ area of base $\times h$

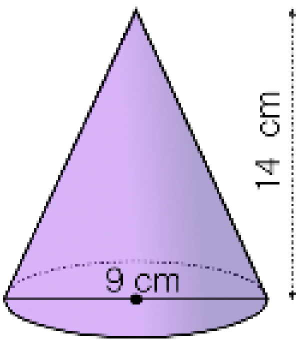
It is likely that Pythagoras' theorem will need to be used to calculate the perpendicular height of the cone or pyramid.

Find the **volume** of each of the following shapes. For the sphere and cone, write your answers (a) in exact form and (b) rounded to 2 decimal places.

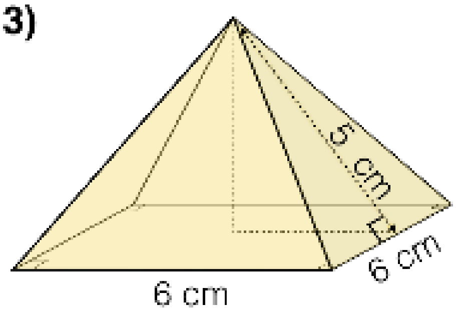
1)



2)



3)



Similar shapes

Websites, resources & activities

- www.resourceaholic.com
- www.bossmaths.com
- Www.tes.com

Why we think this is a useful activity

Allow students to draw some shapes and work out themselves what happens to the area when the sides are doubled, tripled etc.

Use isometric paper to draw some cuboids and explore with volume too.

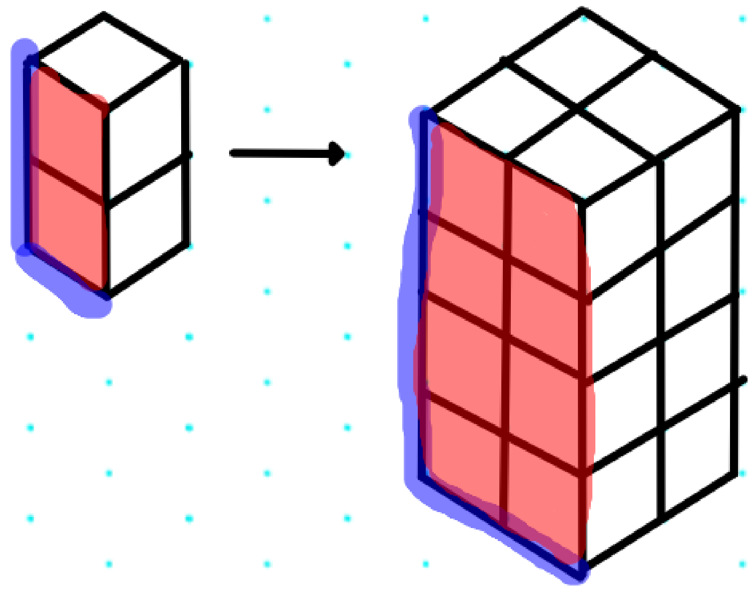
Misconceptions

⇒ If you double the length of the sides then the area will also double.

⇒ If you double the length of the sides then the volume will double.

⇒ Identifying the scale factor and knowing whether it is k, k^2 or k^3

Similar shapes



length of sides x 2

area of face x 4

volume of cuboid x 8

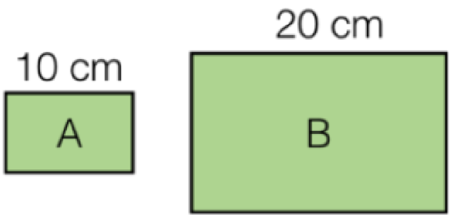
If the scale factor of enlargement is k then,

Length scale factor	k
Area scale factor	k^2
Volume scale factor	k^3

Note: Perimeter counts as a length

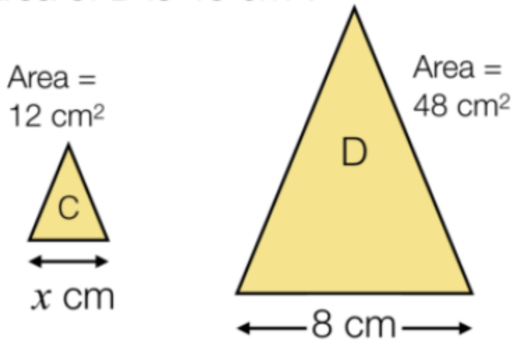
Area scale factor questions

Here are two similar rectangles.
The perimeter of A is 32 cm and its area is 60 cm².



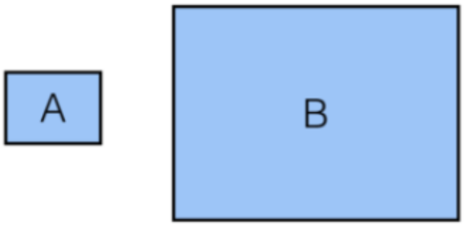
a) Find the perimeter of rectangle B.
b) Find the area of rectangle B.

Here are two similar triangles.
The area of C is 12 cm² and the area of D is 48 cm².



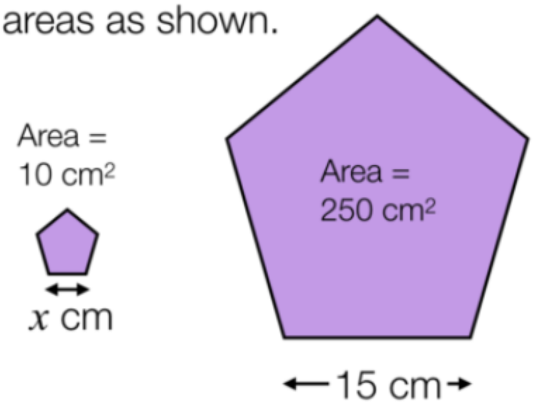
c) Work out the value of x.

Here are two similar rectangles.
The perimeter of A is 15 cm and the perimeter of B is 45 cm.



a) Given that the area of rectangle A is 12 cm², find the area of rectangle B.

Here are two similar shapes, with areas as shown.



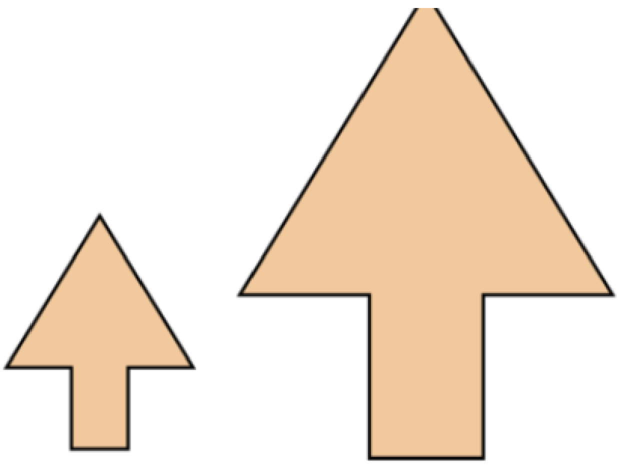
b) Work out the value of x.

Two similar shapes are shown.

The smaller shape has an area of 8 cm².

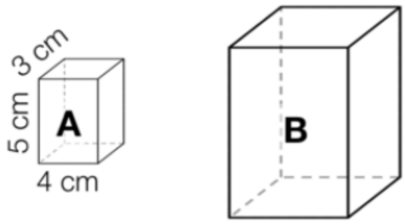
The larger shape has an area of 32 cm² and a perimeter of 32 cm.

What is the perimeter of the smaller shape?



Volume scale factor questions

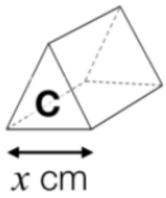
Cuboid B is an enlargement of cuboid A with scale factor 2.

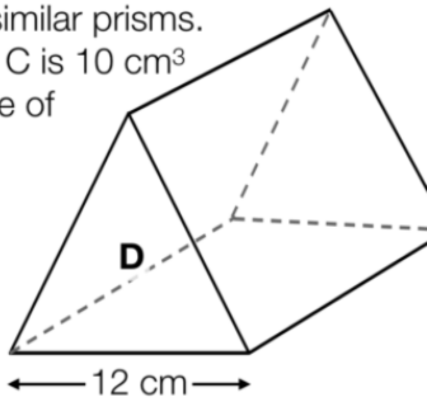


a) Compare the volumes of the two cuboids.

b) Compare the surface areas of the two cuboids.

Here are two similar prisms. The volume of C is 10 cm³ and the volume of D is 270 cm³.

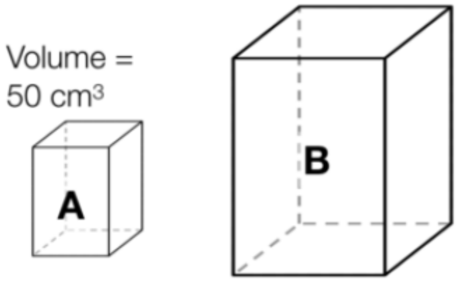




c) Work out the value of x.

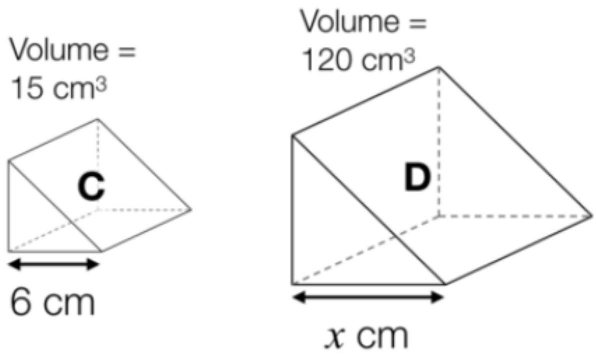
d) Given that the surface area of D is 693 cm², work out the surface area of C.

Cuboids A and B are similar. Cuboid B is 3 times taller than A.



a) Work out the volume of cuboid B.

Prism D is an enlargement of prism C.

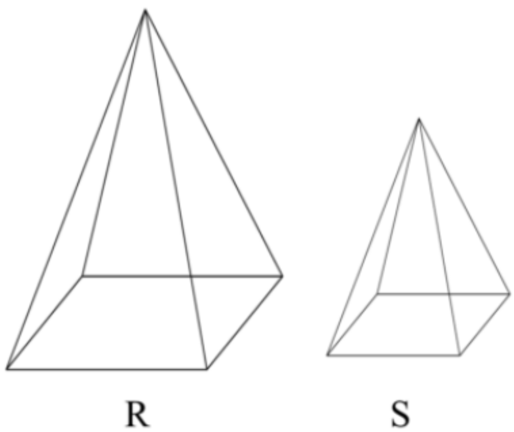


b) Work out the value of x.

R and S are mathematically similar square-based pyramids. The surface area of R is 625 cm² and the surface area of S is 100 cm².

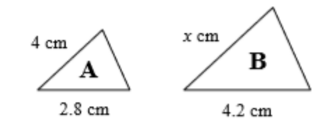
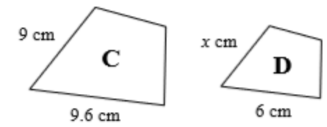
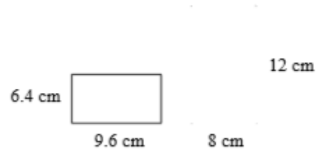
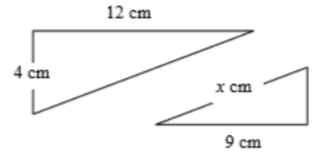
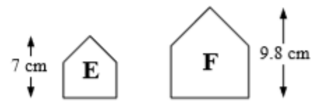
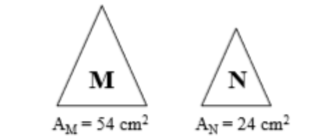
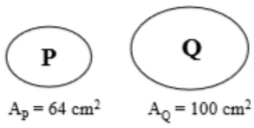
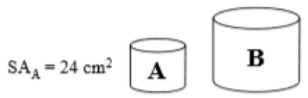
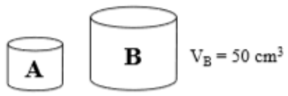
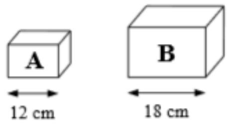
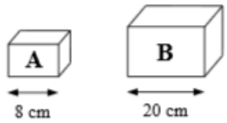
a) The length of one side of the square base of R is 12.5 cm. What is the length of one side of the square base of S?

b) The volume of S is 56 cm³. Work out the volume of R.



Increasing difficulty

questions

A1 Shape **A** is similar to shape **B**  Work out the value of x.	**A2** Shape **C** is similar to shape **D**  Work out the value of x.	**A3** Do some calculations to work out if the rectangles are mathematically similar. 	**A4** The two triangles are similar  Work out the value of x.
B1 Shape **E** is similar to shape **F** The area of **E** is 30 cm^2  Calculate the area of **F**.	**B2** Shape **G** is similar to shape **H** The area of **G** is 210 cm^2  Calculate the area of **H**.	**B3** Shape **M** is similar to shape **N** The height of **M** is 12 cm.  Calculate the height of **N**.	**B4** Shape **P** is similar to shape **Q** The width of **Q** is 14 cm.  Find the width of **P**.
C1 Cylinders **A** and **B** are similar Cylinder **B** is 1.6 times as high as cylinder **A**.  Calculate the surface area of **B**.	**C2** Cylinders **A** and **B** are similar Cylinder **B** is 1.4 times as high as cylinder **A**.  Calculate the volume of **A**.	**C3** Cuboids **A** and **B** are similar The volume of **A** is 250 cm^3.  Calculate the volume of **B**.	**C4** Cuboids **A** and **B** are similar The surface area of **B** is 1000 cm^2.  Calculate the surface area of **A**.

Each question gets slightly more difficult and so eases the students into more challenging problems. These resources were found on TES titled Maths4everyone and there are resources like this for most mathematical topics.